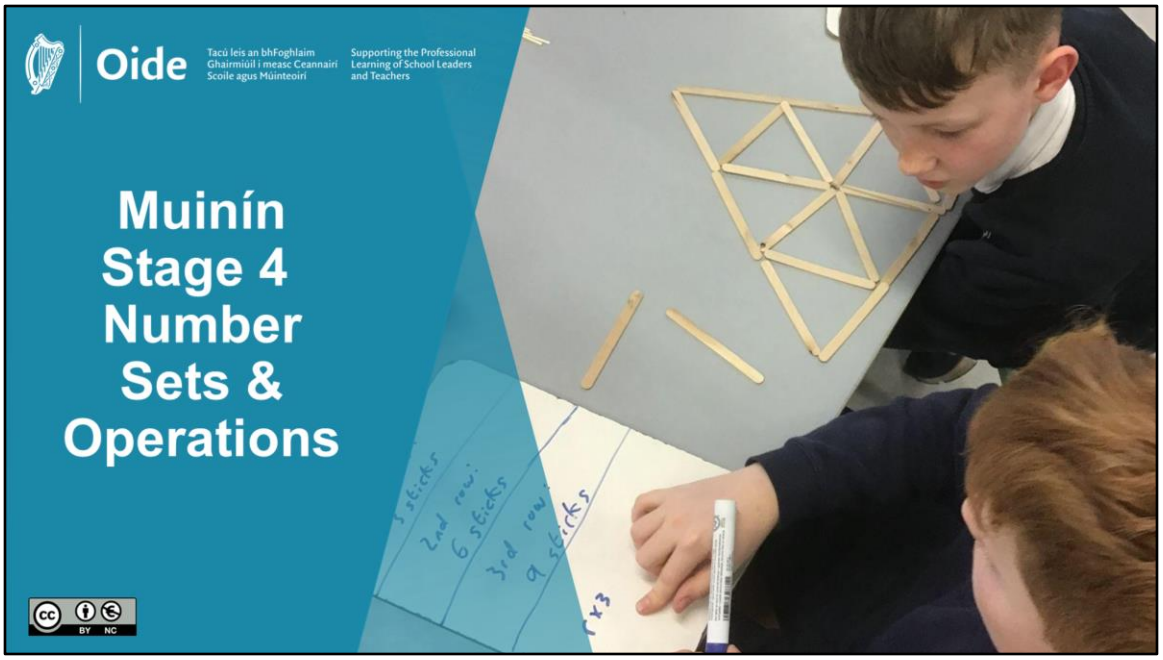
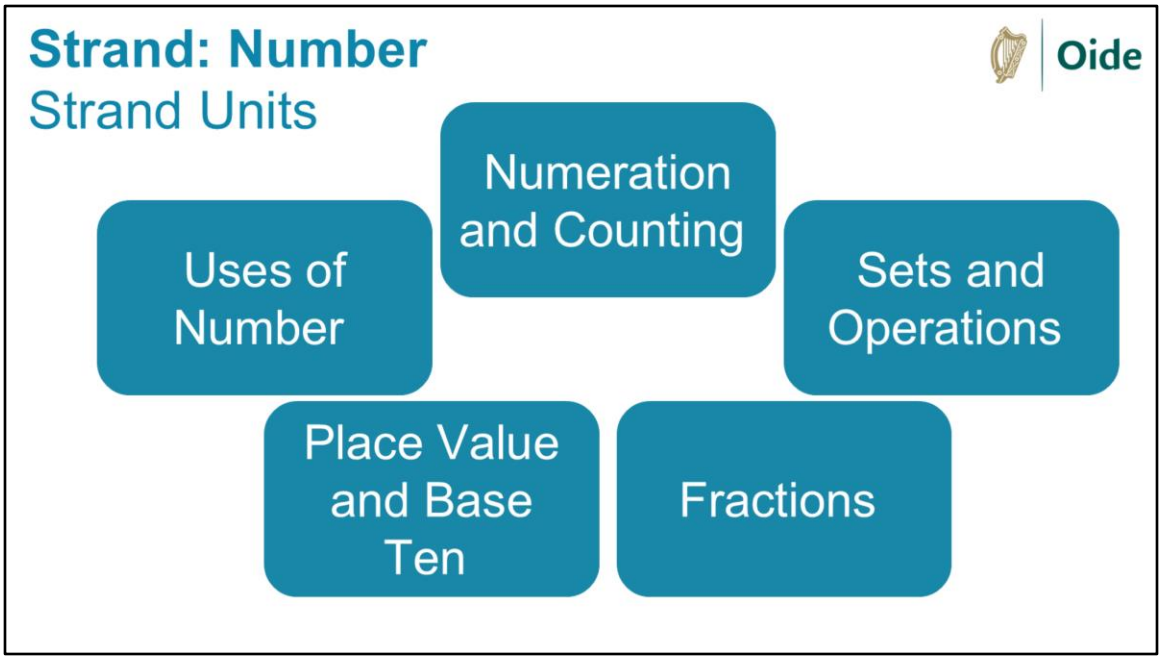


Purpose of slide:
Introductory slide for presentation of Stage 4 Number- Sets and Operations.



Purpose of slide:
To provide teachers with an overview of the Number Strand.

Notes for teachers:

- Go to page 22 of the Primary Mathematics Curriculum.
- There are 5 strand units within the strand of Number in the primary mathematics curriculum. There are uses of number, numeration and counting, place value and base ten, sets and operations and fractions.
- Notice:
    - Uses of number is in stage 1 only.
    - Numeration and counting is stage 1 and 2 only.
    - Although numeration and counting strand unit is not in Stage 4 - we encourage teachers to continue to develop children's sense of number with counting in Stage 3 and 4 – fractions/decimals/negative numbers as counting is the foundation of all number sense development.
    - Place value and base ten, sets and operations and fractions are there for all stages.

Strand Unit: Sets and Operations

Progression across the stages

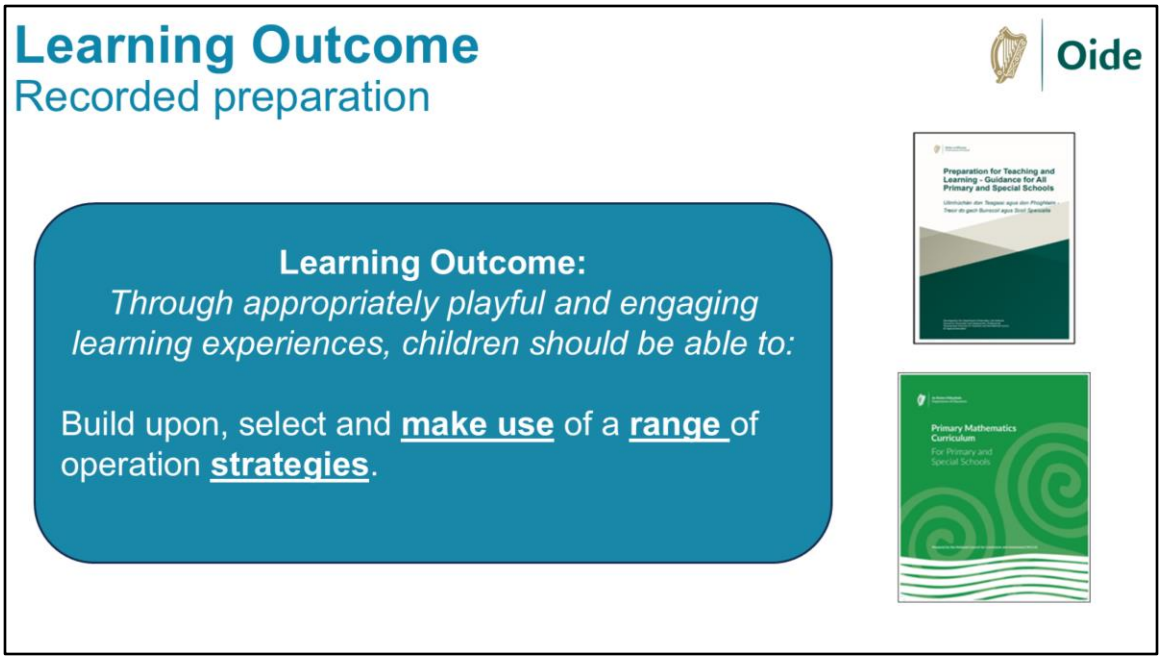
Oide

Learning Outcomes for Sets and Operations Strand Unit			
Stage 1: Junior and senior infants	**Stage 2:** First and second classes	**Stage 3:** Third and fourth classes	**Stage 4:** Fifth and sixth classes
Through appropriately playful and engaging learning experiences, children should be able to			
recognise and understand what happens when quantities (sets) are **partitioned** and **combined**	select, make use of and represent **a range of addition** and **subtraction strategies.**	**understand** and **apply flexibly** the four operations; and the **relationships** between operations.	build upon, select and make use of a **range of operation strategies.**

Purpose of slide:
To explore the progression across the stages in the strand unit Sets and Operations.

Notes for teachers:

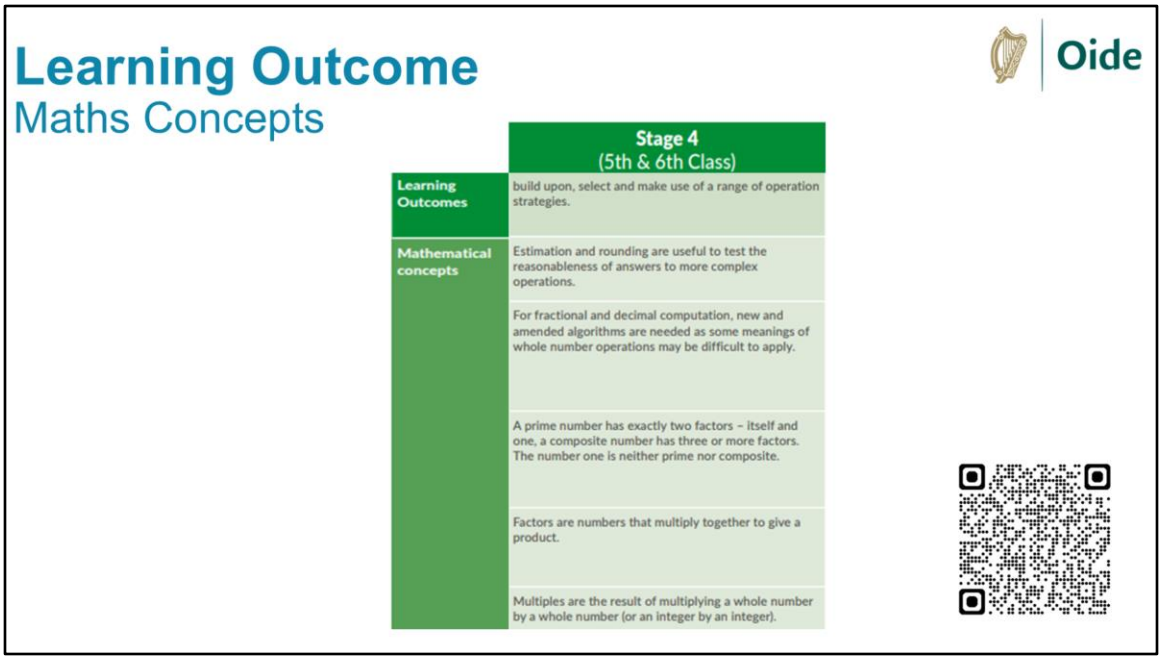
- Notice the progression along the stages.
- Note how language, knowledge and skills are developed from stages 1 to 4.
- Knowledge of progression is necessary so that we can adapt and extend our teaching based on the knowledge we have of the children in front of us.
- Looking at the learning outcomes we can see how each stage builds upon the last, fostering a rich understanding of sets and operations and its mathematical significance.
- The TIMSS 2019 report highlights that a pupil's ability to perform well in number-related tasks often correlates with their understanding of sets and operations, as these foundational concepts are essential for grasping more complex mathematical ideas.



Purpose of slide:
To highlight the learning outcome as the starting point for preparation for teaching and learning.

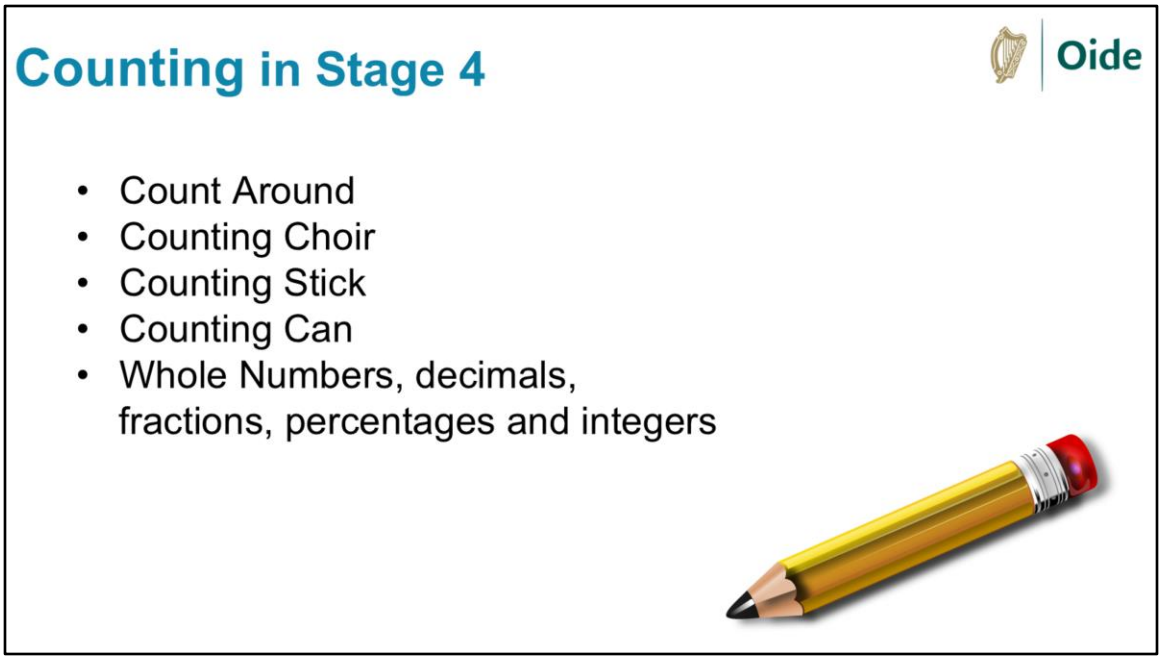
Notes for teachers:

- This is the learning outcome for Stage 4 Number- Sets and Operations.
- Learning outcomes are broad in nature. They are the big mathematical ideas that pupils work towards over a 2-year period.
- When working with learning outcomes it is useful to break down the learning outcome into areas of focus using the maths concepts (see next slide).
- For Stage 4 Sets and Operations the pupils will *build upon, select and make use of a range of operation strategies.*
- Consider the key language, knowledge and skills within this learning outcome.



Purpose of slide:
To highlight the Maths Concepts for Stage 4 Number – Sets and Operations.

Notes for teachers:
•The Maths Concepts are the key mathematical ideas that underpin each learning outcome.
•The Maths Concepts may be useful in identifying a Focus of New learning when preparing for teaching and learning.
•Take a few moments to explore the Learning Outcomes and the Maths Concepts on the NCCA Maths Toolkit by using the QR code above.



Purpose of slide:

To highlight counting as the first step in mental calculations.

To demonstrate some counting activities for stage 4 division.

Notes for teachers:

- Counting is the foundation of the development of number sense. It is the first step in mental calculation.
- It is important to develop pupils' flexibility with counting. All pupils will benefit greatly from simple counting activities.
- Many children who struggle with Maths don't have a full grasp of number sequences and can gain in confidence from daily or regular number work.
- Points to note when counting:
    - Different starting points.
    - Skip counting as a way of multiplying through repeated addition. It is recommended to begin skip counting with 2s, 10s and 5s as

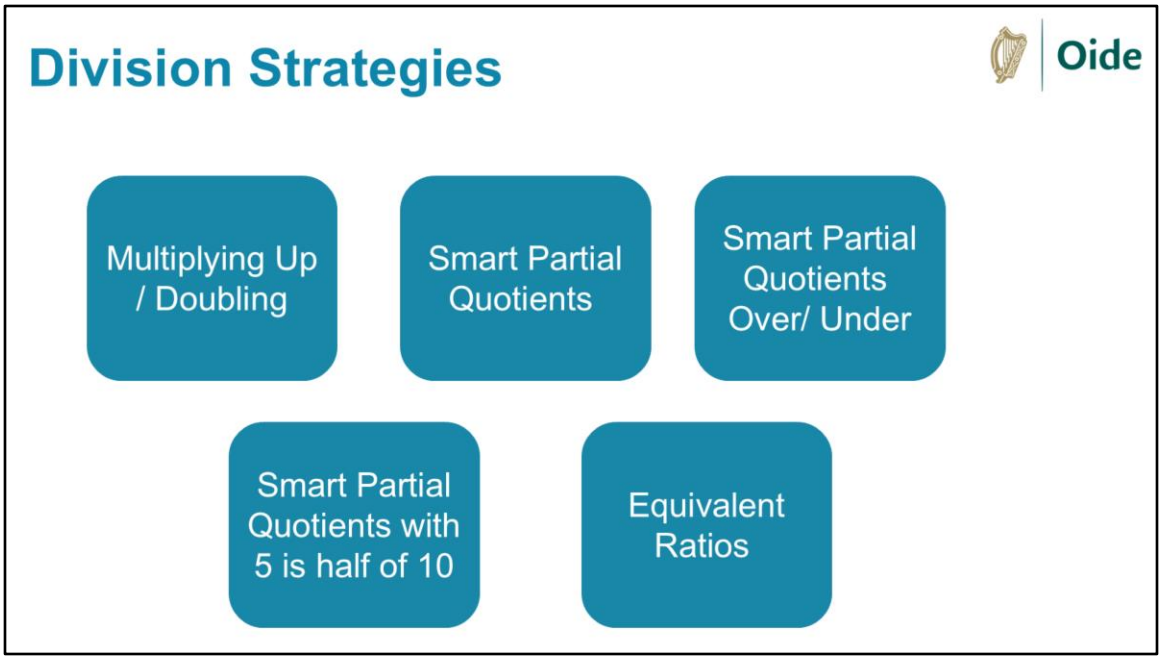
other facts can be derived from these numbers.

- Counting in doubles is also a great way for children to be flexible with numbers as they can then count forwards or backwards from the double.
- When children understand and can count flexibly in 2s, 5s and 10s, they can then break apart those numbers to count in 3s, 4s, 6s,7s, 8s, 9s. For example when children can count in 2s, they can then use this knowledge to count in 3s by doubling the number and adding one more. The same goes for counting in 5s, when children can count in 5s they can then count in 6s by multiplying the number by 5 and adding one more. The same works for counting in nines, when a student can count in 10s, they can multiply any number by 10 and subtract one away to find the answer for example 8 x 9= 8 x10 - 8

- We need children to understand the magnitude of number and see the patterns and relationship within the number sequence.
- As well as counting, another important aspect is learning about numerals - identify, recognise, sequence, order, locate and write numerals.
- Counting is for all stages and should be done daily.
- As students come to know the basic facts in any operation, they progress through three phases (Baroody, 2006)
    1. Counting
    2. Deriving (reasoning strategies on known facts)
    3. Mastery (efficiently produces answers)
- A counting session should have:
    - A lively pace.
    - Enthusiastic participation.
    - 2 or 3 short focused activities.
    - Physical activity.
    - Choral response.
    - Individual response.
- <u>Some Suggested Counting Activities: (</u>These activities can be done with any number range and when counting in single numbers or multiples).
    - Counting choir – start with choral counting, move onto parts of the choir.
    - Count around model bridging unusual ranges of numbers (forwards and backwards) 73 – 65.
    - Counting Can (1s, 5s, 10s etc.).

- Counting stick - https://mathsbot.com/manipulatives/countingStick or https://bossmaths.com/countingstick/

Resource:
Learning experiences that help to develop counting can be found here
Counting Stage 3 and 4
https://pmc.oide.ie/resources/supportmaterialsforschools/

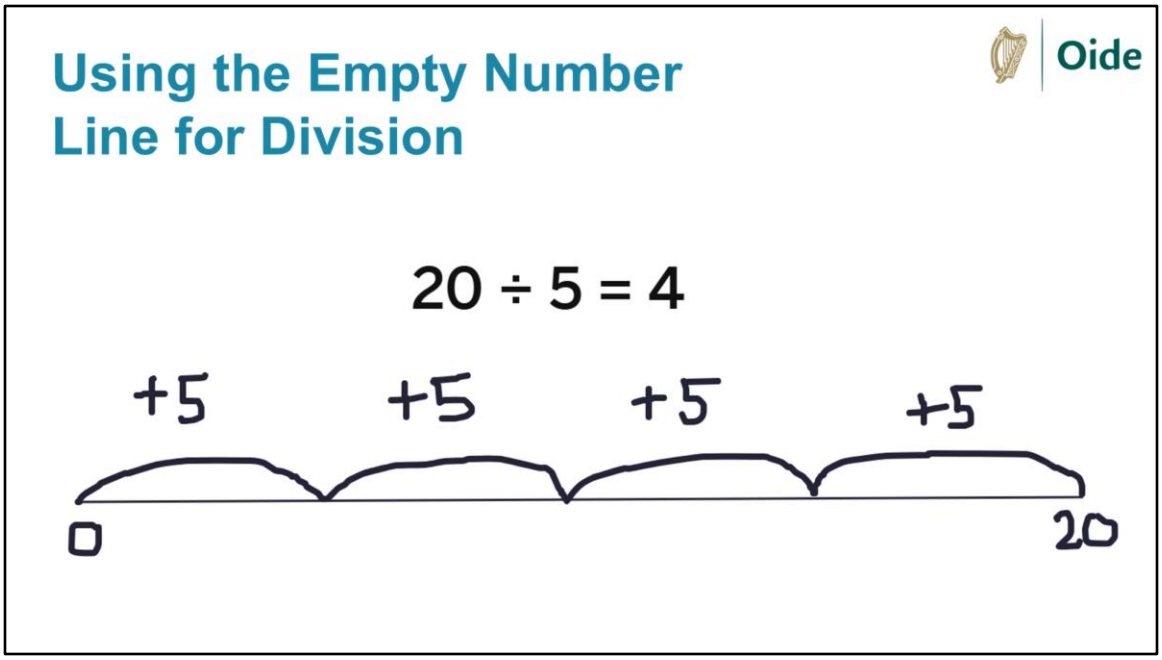


Purpose of slide:
To provide an overview of a range of division strategies.

Notes for teachers:

- These are a number of division strategies that the children can explore:
    - Multiplying up/Doubling
    - Smart Partial Quotients
    - Smart Partial Quotients over/under
    - Smart Partial Quotients with 5 is half of 10
    - Equivalent Ratios
- Naming the titles of the strategies is not the most important thing. More importantly is that the children are exposed to/understand the strategy and get opportunities to explore and engage with the strategy.

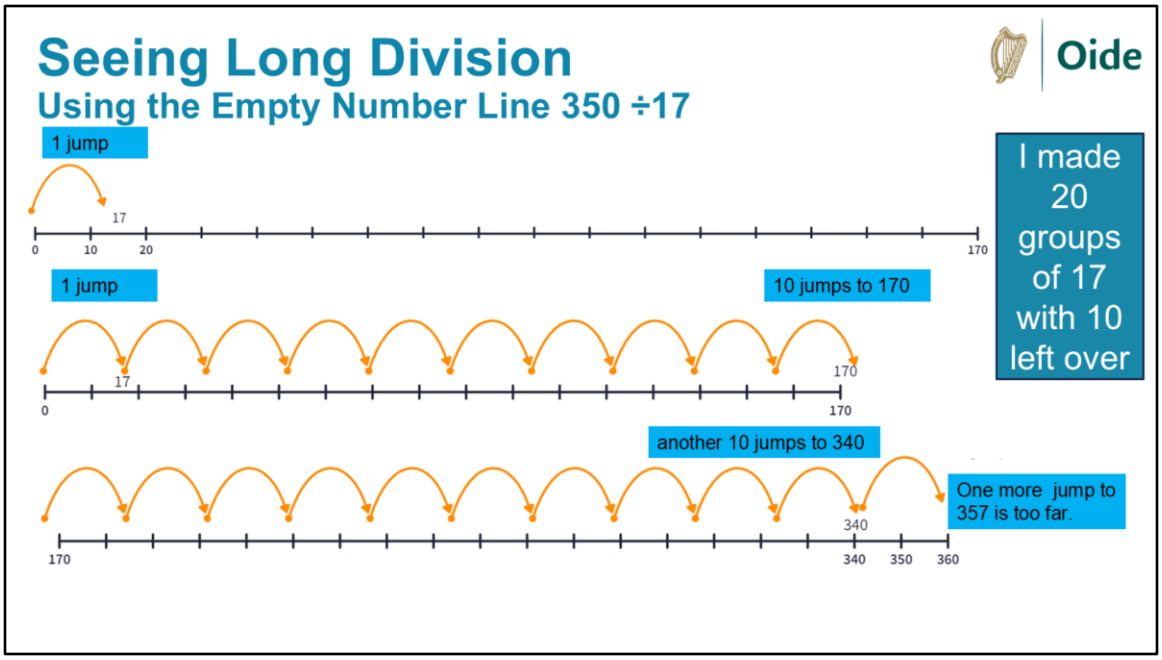
- It is possible to combine a couple of strategies to attack/solve a division problem also.
- Provide children with agency. Encouraging them to choose a strategy that makes sense to them is important.
- Give them ample time and opportunities to explain how they did it/what strategy they used and why they chose it. This is central to developing maths talk.



Purpose of slide:
To demonstrate how an empty number line can be used in division.

Notes for teachers:

- An empty number line is a blank or unmarked line that's used as a flexible tool to help students understand numbers and their relationships — especially for teaching addition, subtraction, place value, and number sense.
- Key Features:
    - It has no numbers or tick marks to start with.
    - Students or teachers add numbers and marks as needed.
    - It encourages mental math and different strategies for solving problems.
- In this slide, how many jumps of five are in 20?



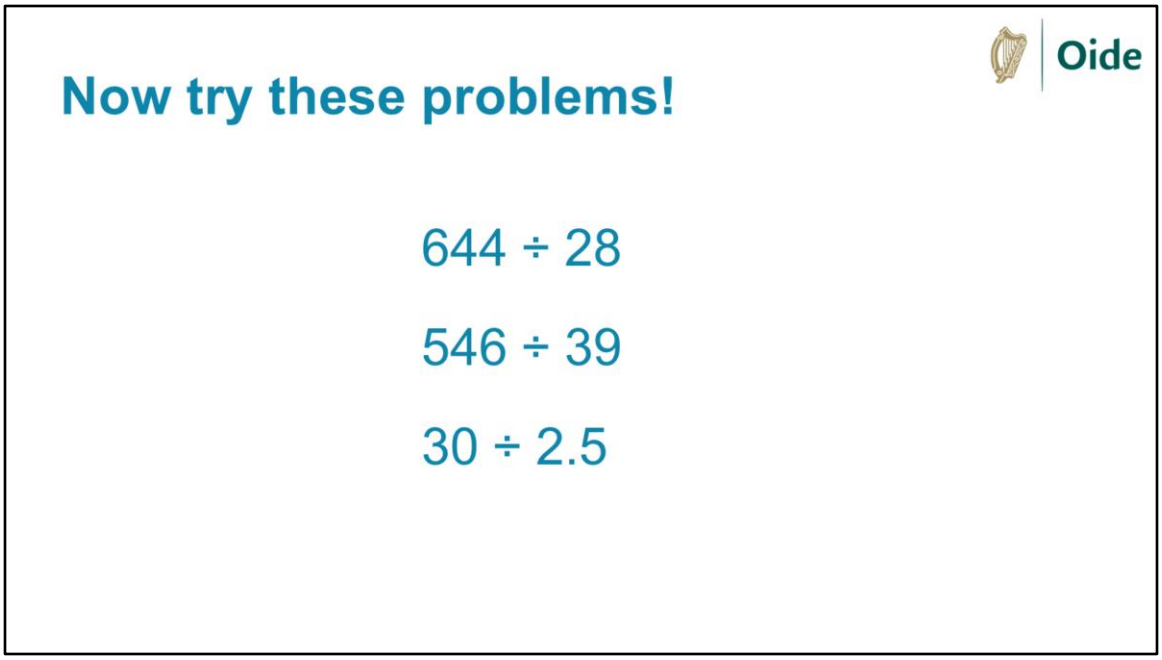
Purpose of slide:
To show how empty line could be used for bigger numbers.

Notes for teachers:

- What is division? What language is necessary for children to understand what division is?
    - Division is the inverse of multiplication. Division is creating "groups of" by sharing and finding out how many "times" a number might be found in another, whether evenly or unevenly. Using language like 17 "goes into" 350 is confusing for many children because they cannot see the link to multiplication.
- Some children believe they are unable to "do" long division because they are following the rules or the steps of the procedure without the conceptual understanding behind those step i.e Why am I doing this? For many, it does not make sense to them.
- In this task, children use their knowledge of multiplying by 10 so they don't have to skip count in 17's all the way to 340.
- They know 10 x 17 =170. Building on that, another 10 x 17 bring us up to 340. Another 17 is too far so we can find the difference between 340 and

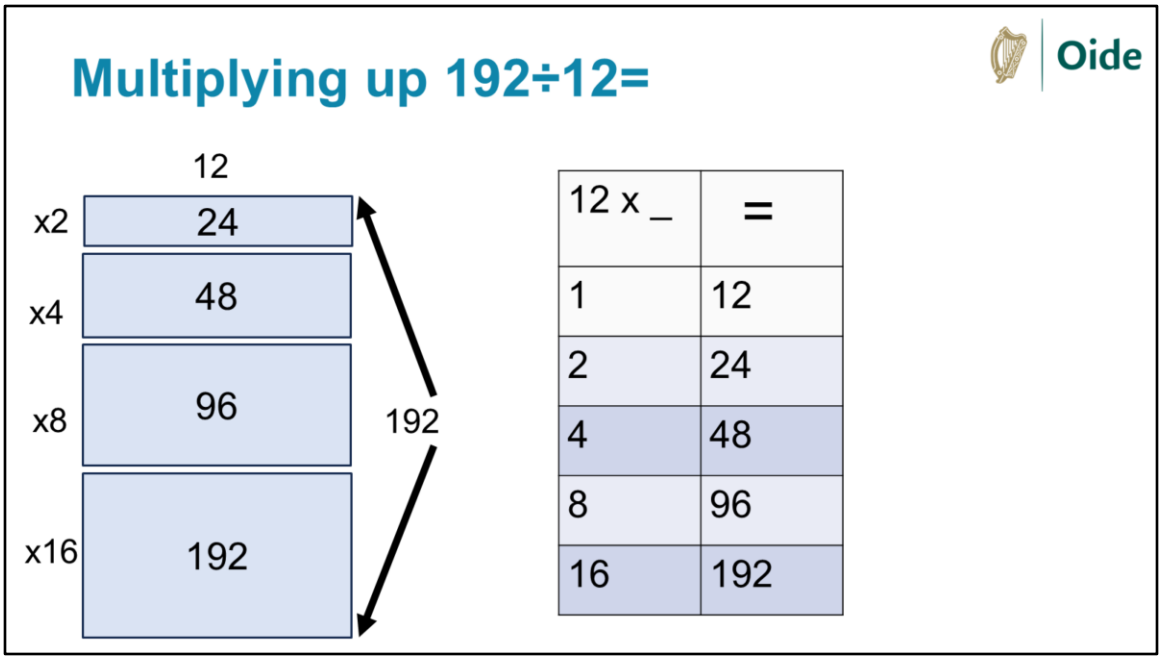
350. *“ I made 20 groups of 17 with 10 left over”.*

- Skip counting forward or backwards in multiples or in groups of 10 of those multiples, on an empty number line, helps children to visualize and make connections between the numbers in the problem.



Purpose of slide:
To provide the opportunity to use the Empty Number Line (ENL) to solve division questions.

Notes for teachers:
- Try using the strategy and reflect on how you think this may work in your class.
- If the children have never used the ENL before, it will be necessary to model it first.



Purpose of slide:
To introduce multiplying up as a strategy.

Notes for teachers:

- Multiplication and Division are often taught separately, with multiplication preceding division. However, division and multiplication are inverse operations. Every multiplication calculation can be replaced by equivalent division calculations and vice versa. Therefore, it is important to combine multiplication and division soon after multiplication has been introduced in order to help pupils see how they are related. The multiplying up strategy is based on the distributive property. *Although not one of the four major division strategies, students may choose to think about division as multiplication and make use of doubling the divisor until they get to or close to the dividend.*
- In this problem, 192 divided by 12, the child is going to chunk known facts.
    - The child knows that 12 x 2 is 24
    - They then double this to get to 48 which is equal to 4 times 12
    - They again double 48 to get 96 which is the same as 12 times 8
    - Finally they double it again to get to 192 which is the same as 12

times 16. Using the distributive property the child can show that 192 divided by 12 is equal to 16.

- Here we also see how a child can use the same strategy using a table. Children can and will use a combination of strategies to solve problems.
- Multiplying up works nicely here but it can also get us very close to answers in other examples .
- If this question was 195 divided by 12, what will the pupil do?

Now try these problems!

Oide

180 ÷ 15

448 ÷ 32

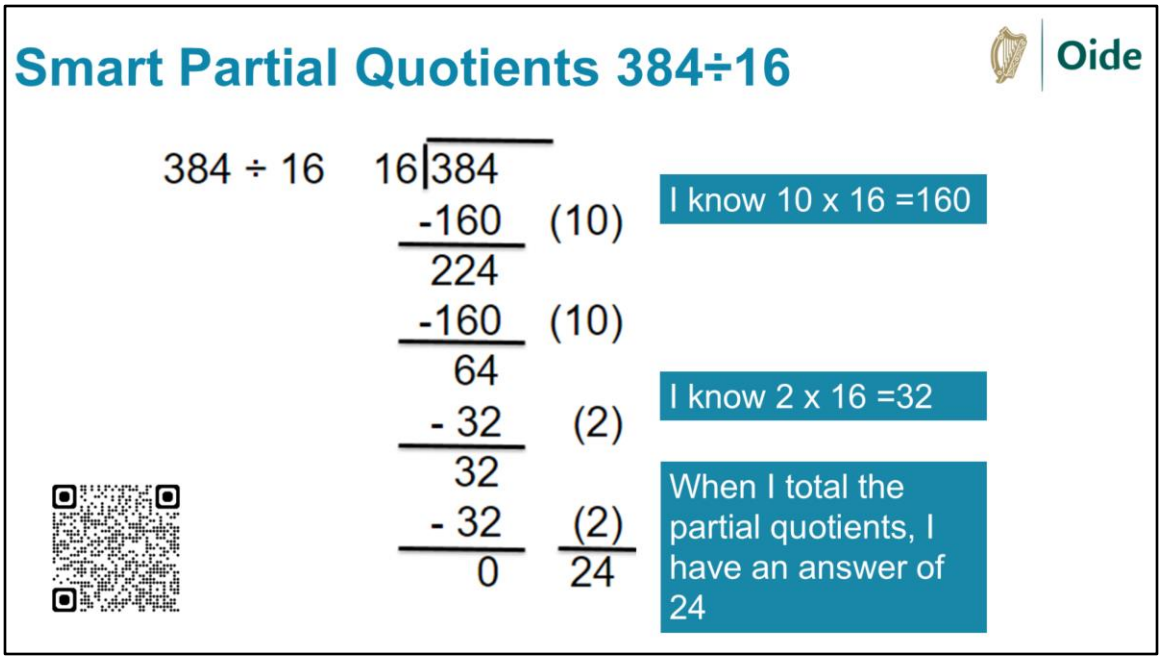
648 ÷ 24

Sample of Multiplying Up

12 x _	=
1	12
2	24
4	48
8	96
16	192

Purpose of slide:
To provide the opportunity to practice division using multiplying up strategy.

Notes for teachers:
- Try using the strategy yourself and reflect on how you think this may work in your class.



Purpose of slide:
Introduce the smart partial quotients strategy for division.

Notes for teachers:

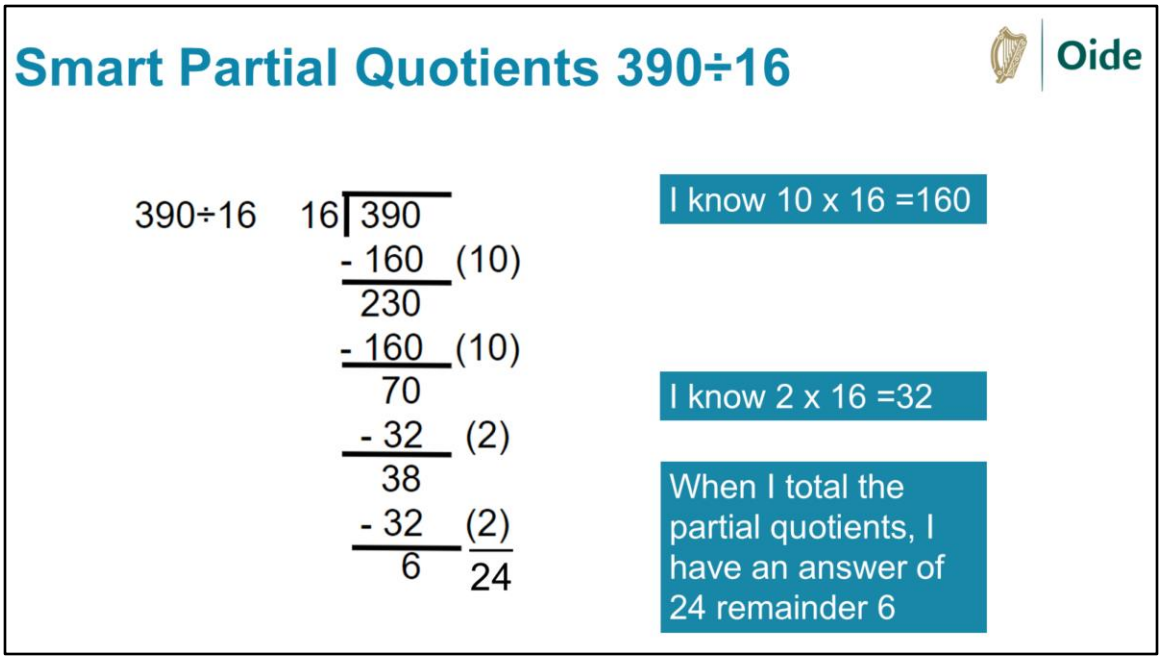
- This strategy involves breaking down a division problem into smaller, more manageable parts (partial quotients) by focusing on easier multiples, such as tens, fives, and twos, of the divisor. By progressively subtracting these partial quotients from the total, the final quotient is reached in a step-by-step manner
- Children need to use their number sense and have confidence in using simple number facts to help them solve this division problem.
- Children's knowledge of the number 10 is vital.
- Division is the inverse of multiplication. Children can use their known facts to get started on the problem.
- Elicit from the pupils, number facts that they already know about 16, for example, 10 x 16 = 160 and 2 x 16 = 32
- This strategy maintains place value and mathematically correct information for students. It allows them to work their way toward the quotient by using friendly multipliers such as tens, fives, and twos without having to

immediately find the largest quotient. As the pupil chooses larger multipliers, the strategy becomes more efficient.

- When learning the procedure for the standard algorithm, pupils are often told that 16 cannot go into 3 (300) which is incorrect; 16 can divide into 3, but it would result in a fraction. With the Partial Quotients strategy, the "3"maintains its value of 300 and can certainly be divided by 16.
- As the pupil's work, they keep track of the partial quotients by writing them to the side of the problem. When the problem is solved, the partial quotients are totalled, and the final answer is written over the dividend. This example demonstrates using friendly 10s and 2s to solve the problem. As the 10s and 2s are recorded to the side of the problem, they represent 10 x 16 and 2 x 16.

Resources:

Further information about the Smart Partial Quotients and how you can help pupils develop this strategy is available in the Exploring Operations Strategies document via the QR code or this link https://pmc.oide.ie/resources/supportmaterialsforschools/



Purpose of slide:
Working with remainders using the Smart Partial Quotients.

Notes for teachers:

- The same approach can be used as with the example in the previous slide.
- With practise, the pupil may see that they can combine the 2 tens and 2 twos as they work through the multiples, reducing the number of steps they need to take.

  16 x 20 = 320; 16 x 4 = 64; 320 + 64=384 390-384= 6
- The solution is 24 remainder 6

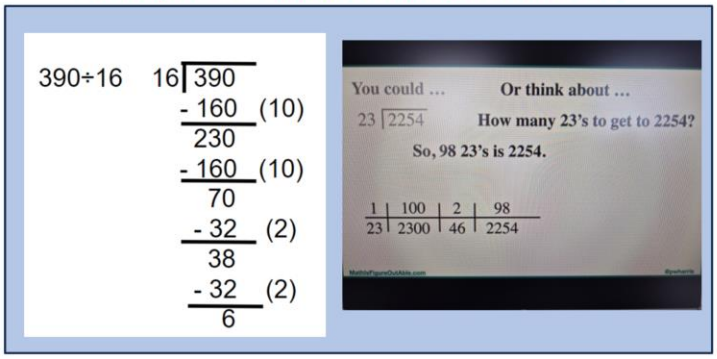
Now try these problems

953 ÷ 31

3624 ÷ 24

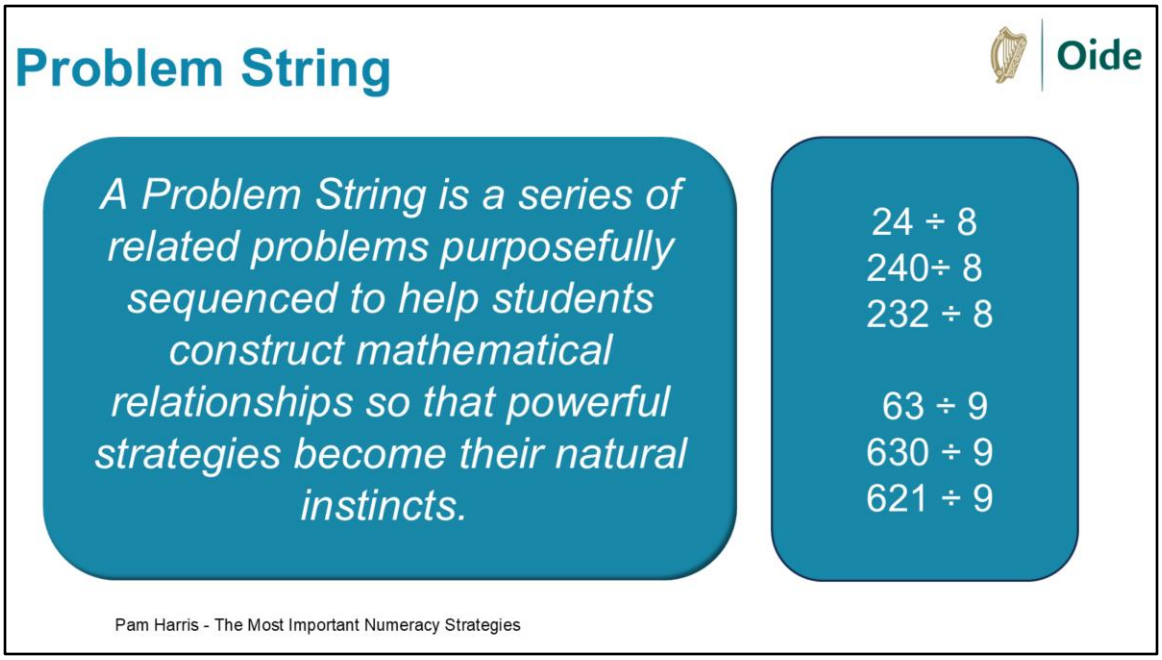
1415 ÷ 12

Sample of Smart Partial Quotients



Purpose of slide:
To provide opportunities to practice division using smart partial quotients.

Notes for teachers:
Note how the 2nd image uses a ratio model to work out partial quotients.



Purpose of slide:
To introduce the use of problem strings.

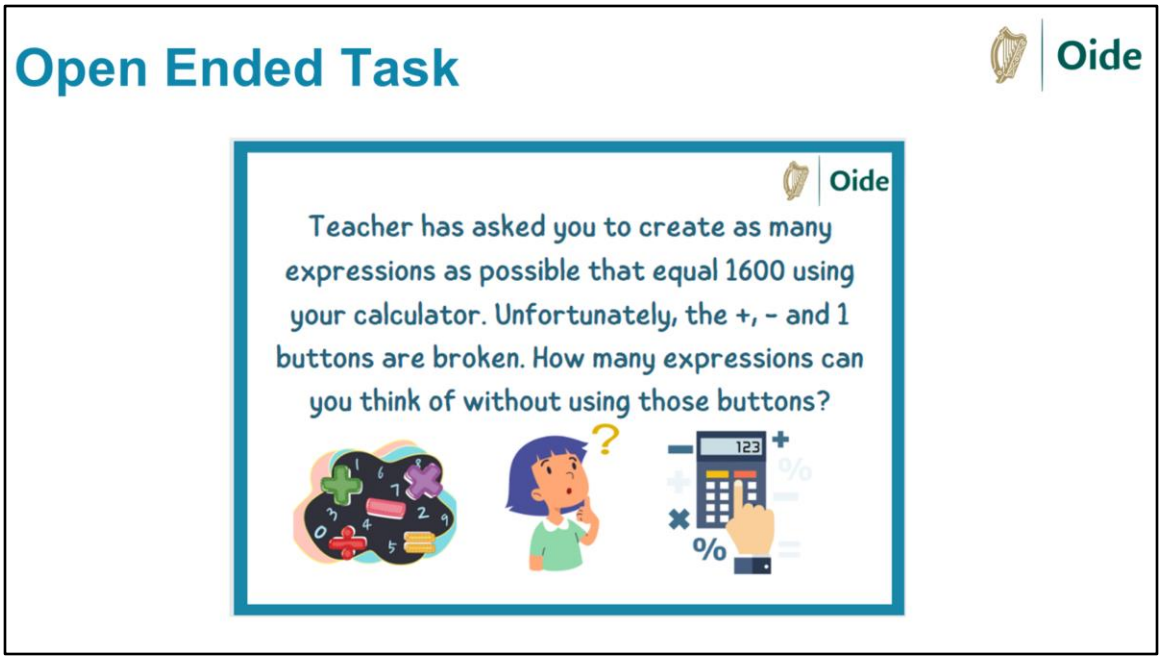
Notes for teachers:

- A Problem String is a powerful teaching tool based on the idea that learning mathematics is about constructing relationships and connections.
- A Problem String is a series of related problems purposefully sequenced to help pupils construct mathematical relationships so that powerful strategies become their natural instincts. This powerful teaching tool is *designed* to help pupils mentally construct mathematical relationships. In this mini-lesson structure, teachers and pupils interact to construct important mathematical strategies, models, and concepts.
- The power of a Problem String lies in the carefully crafted conversation as pupils solve problems one at a time and the teacher makes pupils thinking visible and draws out important connections and relationships.
- How might it look and sound like in the classroom?

- What is 24 ÷ 8? Give pupils time to think. Model-represent on a number line (or ENL) “Even though none of you are writing, I’ll just show 24 ÷ 8 on a number line”.
- What is 240÷ 8? Give pupils time to think. “How do you know? Did anyone use the problem before, 24 ÷ 8 , to help them? How? Represent using 24 ÷ 8 to get 240÷ 8 using another strategy”
- What is 232 ÷ 8? Give pupils time to think. “What is 232 ÷ 8? How do you know? Did anyone use the problem, 240÷ 8 , to help them? How?’’

Resource:

- For further information about Problem Strings you might like to visit https://www.mathisfigureoutable.com/blog/what-is-the-difference-between-a-problem-talk-and-a-problem-string
- Pam Harris' Podcast - Math is Figureoutable Episode 71: "What are Problem Strings Anyway?" provides more insights on this method.



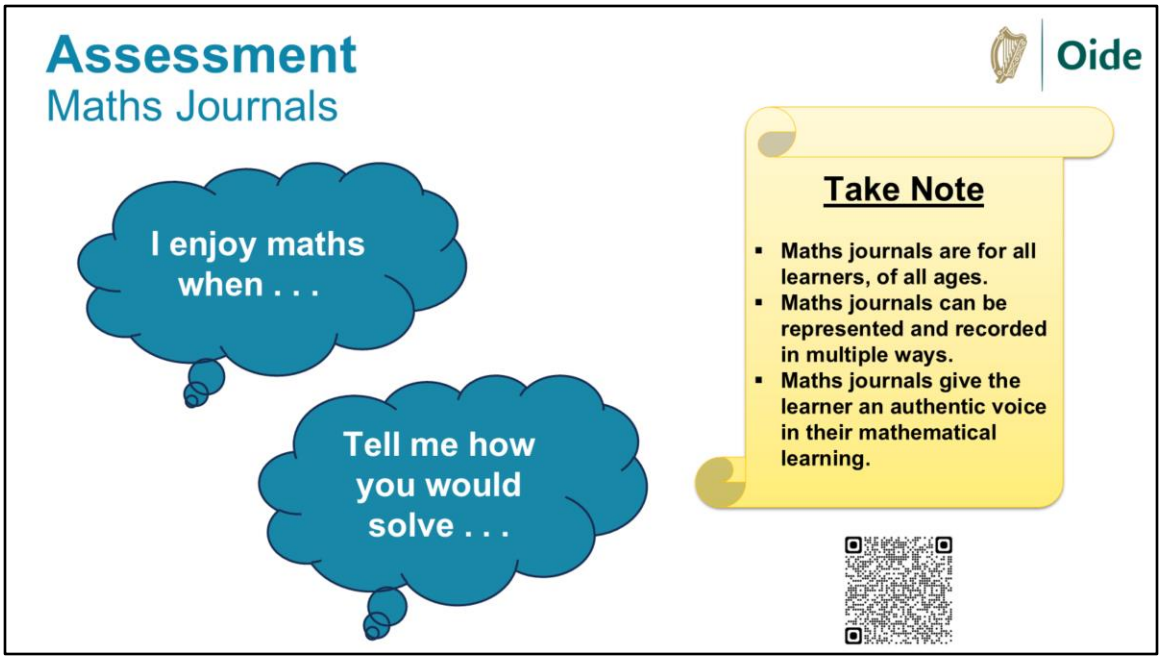
Purpose of slide:
To show an open-ended task where the children can use different division strategies.

Notes for PLL:

- An open-ended task is one where there is a range of 'correct' solutions and/or a range of ways to achieve one or more solutions.
- Using open-ended tasks is one way to encourage playfulness in mathematics and foster a productive disposition to mathematics.
- Open-ended tasks, like this task, provide opportunities for *exploration, investigation, challenge, creativity, choice and independence.*
- Other examples of open-ended tasks will be shared with participants.
- A key aspect of children engaging in open-ended tasks is the follow-up discussions that take place either in a small group or a whole class setting.
- If the emphasis is placed on the generation of different ideas, all children feel that they have something to contribute, and, moreover, learn from the ideas and strategies of their peers.
- Open-ended tasks are also one way of providing for cognitively

challenging tasks in maths.

- When selecting an open-ended task, as with selecting any task, it is important to keep in mind the mathematical point.
- Model talk moves during the follow up discussion e.g. tell me more, can you repeat what …. said.



Purpose of slide:
To provide reflective prompts for use in class.

Notes for teachers:

- Journals are useful for both teachers and learners to assess attitudes, knowledge and skills.
- Children can keep track of their thinking and understanding in the journal.
- Journals can contain general observations about Maths or can be more specific and focus on a particular concept.
- On the slide are two journal prompts which can be used in class. The first one focuses on the child's disposition and can be used across all stand units.
- Journal prompts -
  - I enjoy Maths when .... general prompt to get the children to express their feelings towards maths.
  - Tell me how you would solve e.g. 24 ÷ 8 =; 390÷ 16= Teacher can choose their own problem so that children can demonstrate different division strategies.
- A Maths Journal encourages a child to:

- Reflect on what they have learned and put it in their own words.
- Discuss maths with others (pupil and teacher).
- Identify areas of strength and weakness.
- Evaluate what they have learned.
- If journal writing is done on a regular basis it will help promote mathematical understanding.
- Writing about Maths can have a positive effect that on reducing pupil's Maths anxiety.

Resource:
Further information about Maths Journals available via:
https://pmc.oide.ie/resources/supportmaterialsforschools/