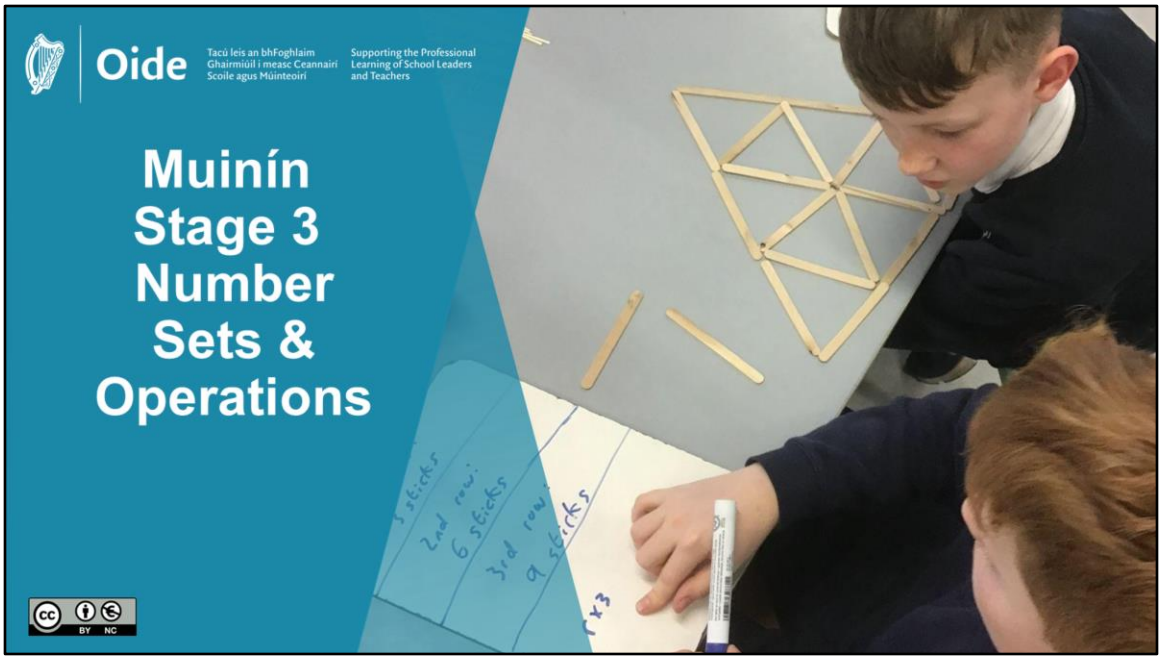
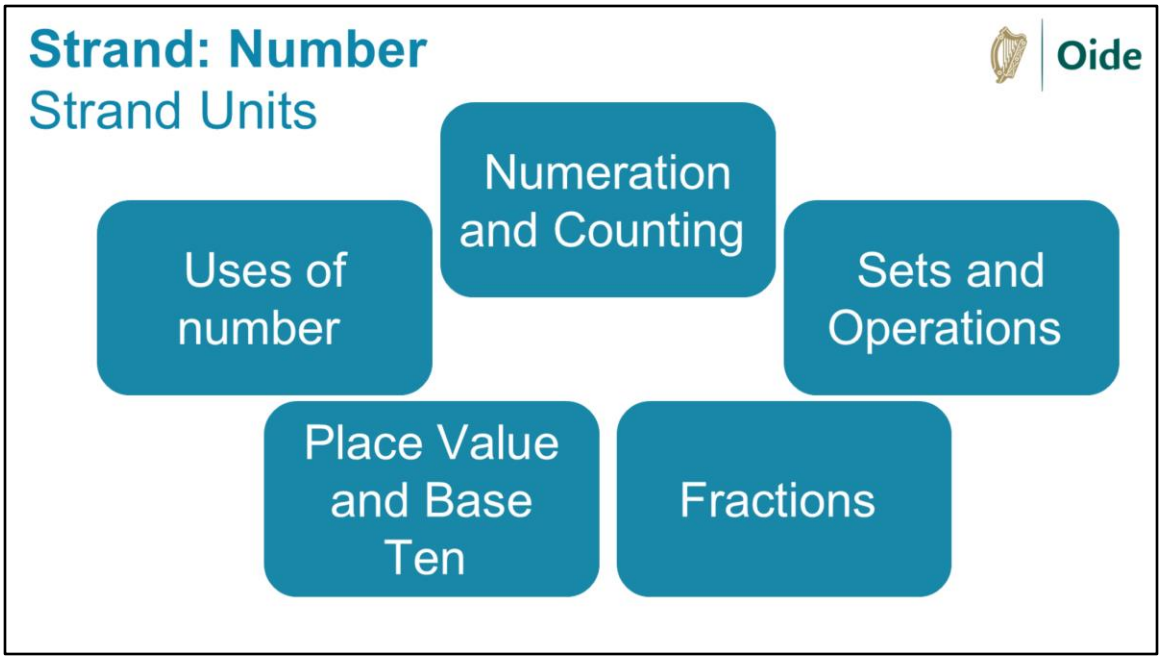


Purpose of slide:
Introductory slide for presentation of Stage 3 Number- Sets and Operations.



Purpose of slide:
To provide teachers with an overview of the Number Strand.

Notes for teachers:

- Go to page 22 of the Primary Mathematics Curriculum.
- There are 5 strand units within the strand of Number in the primary mathematics curriculum. There are uses of number, numeration and counting, place value and base ten, sets and operations and fractions.
- Notice:
    - uses of number is in stage 1 only.
    - numeration and counting is stage 1 and 2 only.
    - place value and base ten, sets and operations and fractions are there for all stages.

Strand Unit: Sets and Operations

Progression across the stages

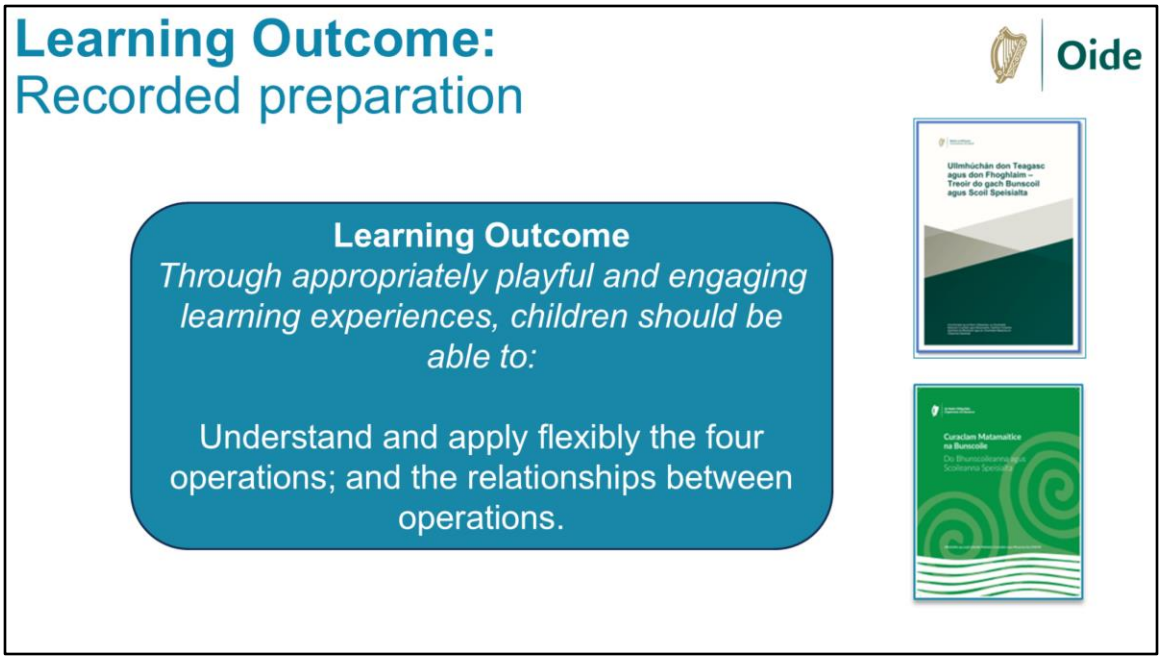
Oide

Learning Outcomes for Sets and Operations Strand Unit			
Stage 1: Junior and senior infants	**Stage 2:** First and second classes	**Stage 3:** Third and fourth classes	**Stage 4:** Fifth and sixth classes
Through appropriately playful and engaging learning experiences, children should be able to			
recognise and understand what happens when quantities (sets) are **partitioned** and **combined**	select, make use of and represent **a range of addition** and **subtraction strategies.**	**understand** and **apply flexibly** the four operations; and the **relationships** between operations.	build upon, select and make use of a **range of operation strategies.**

Purpose of slide:
To explore the progression across the stages in the strand unit Sets and Operations.

Notes for teachers:

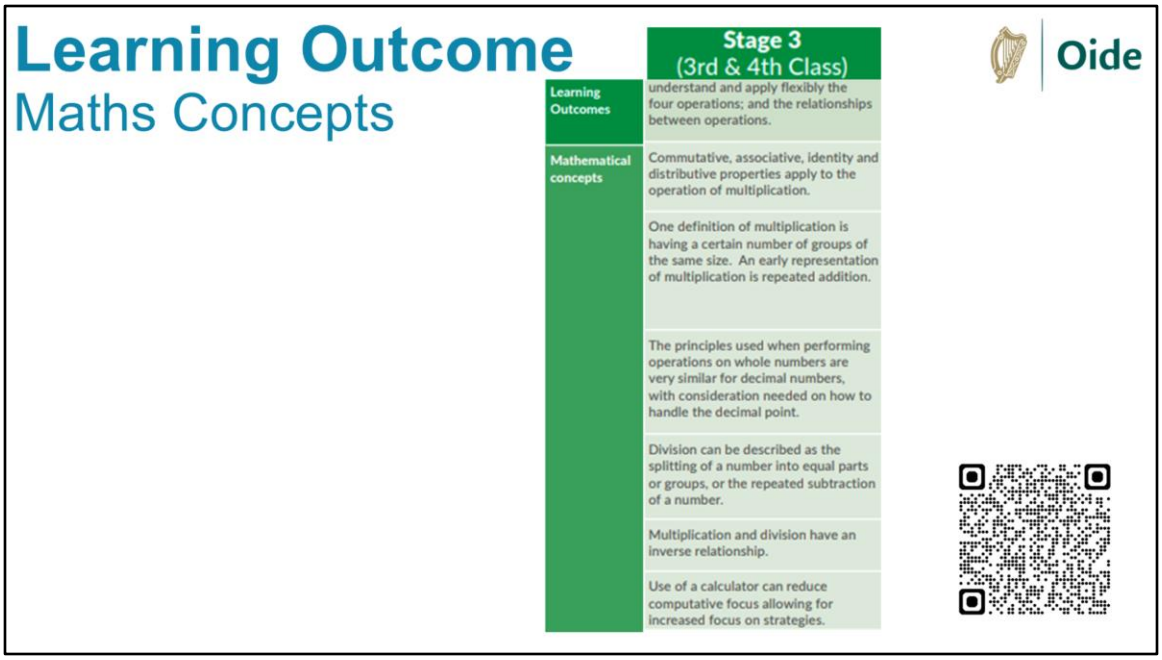
- Notice the progression along the stages.
- Note how language, knowledge and skills are developed from stages 1 to 4.
- Knowledge of progression is necessary so that we can adapt and extend our teaching based on the knowledge we have of the children in front of us.
- Looking at the learning outcomes we can see how each stage builds upon the last, fostering a rich understanding of sets and operations and its mathematical significance.
- The TIMSS 2019 report highlights that a pupil's ability to perform well in number-related tasks often correlates with their understanding of sets and operations, as these foundational concepts are essential for grasping more complex mathematical ideas.



Purpose of slide:
To highlight the learning outcome as the starting point for preparation for teaching and learning.

Notes for teachers:

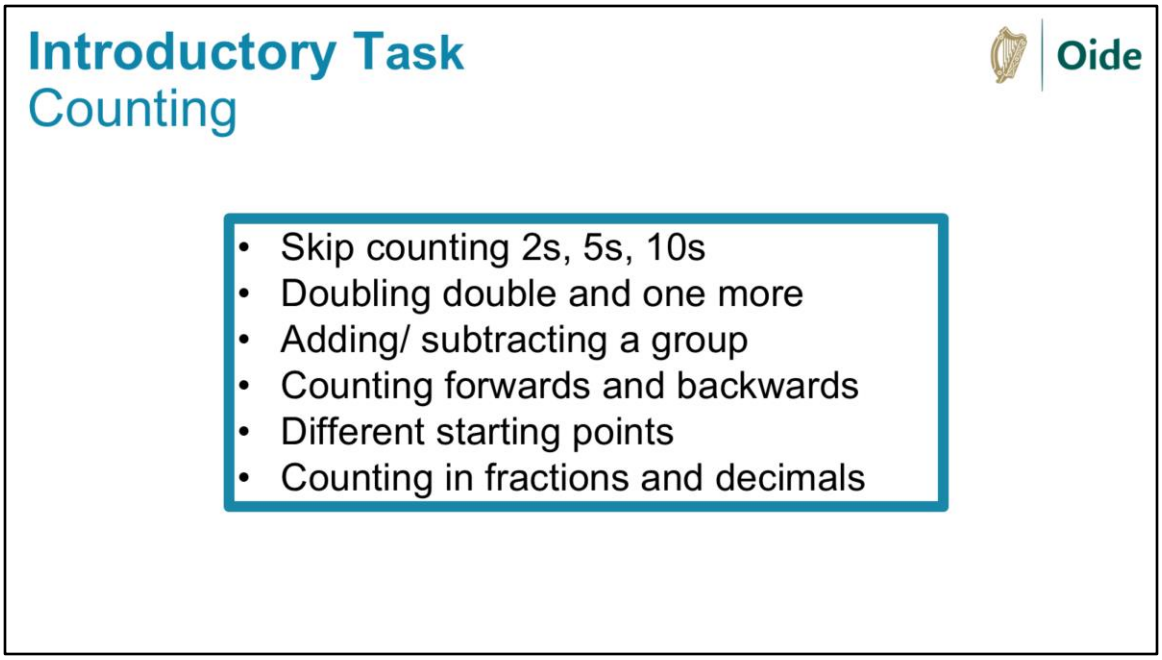
- This is the learning outcome for Stage 3 Number- Sets and Operations.
- Learning outcomes are broad in nature. They are the big mathematical ideas that pupils work towards over a 2-year period.
- When working with learning outcomes it is useful to break down the learning outcome into areas of focus using the maths concepts (see next slide).
- For Stage 3 Sets and Operations the pupils will *understand and apply flexibly the four operations; and the relationships between the operations.*
- Consider the key language, knowledge and skills within this learning outcome.



Purpose of slide:
To highlight the Maths Concepts for Stage 3 Number – Sets and Operations.

Notes for teachers:

- The Maths Concepts are the key mathematical ideas that underpin each learning outcome.
- The Maths Concepts may be useful in identifying a Focus of New learning when preparing for teaching and learning.
- Take a few moments to explore the Learning Outcomes and the Maths Concepts on the NCCA Maths Toolkit by using the QR code above.



Purpose of slide:

To highlight counting as the first step in mental calculations

To demonstrate some counting activities for stage 3 multiplication

Notes for teachers:

- As previously mentioned, counting is the foundation of the development of number sense. It is the first step in mental calculation.
- It is important to develop pupils' flexibility with counting. All pupils will benefit greatly from simple counting activities. Many children who struggle with Maths don't have a full grasp of number sequences and can gain in confidence from daily or regular number work. The three key pillars include:
    - Different starting points.
    - Skip counting as a way of multiplying through repeated addition. It is recommended to begin skip counting with 2s, 10s and 5s as other facts can be derived from these numbers which you will see

in this workshop.

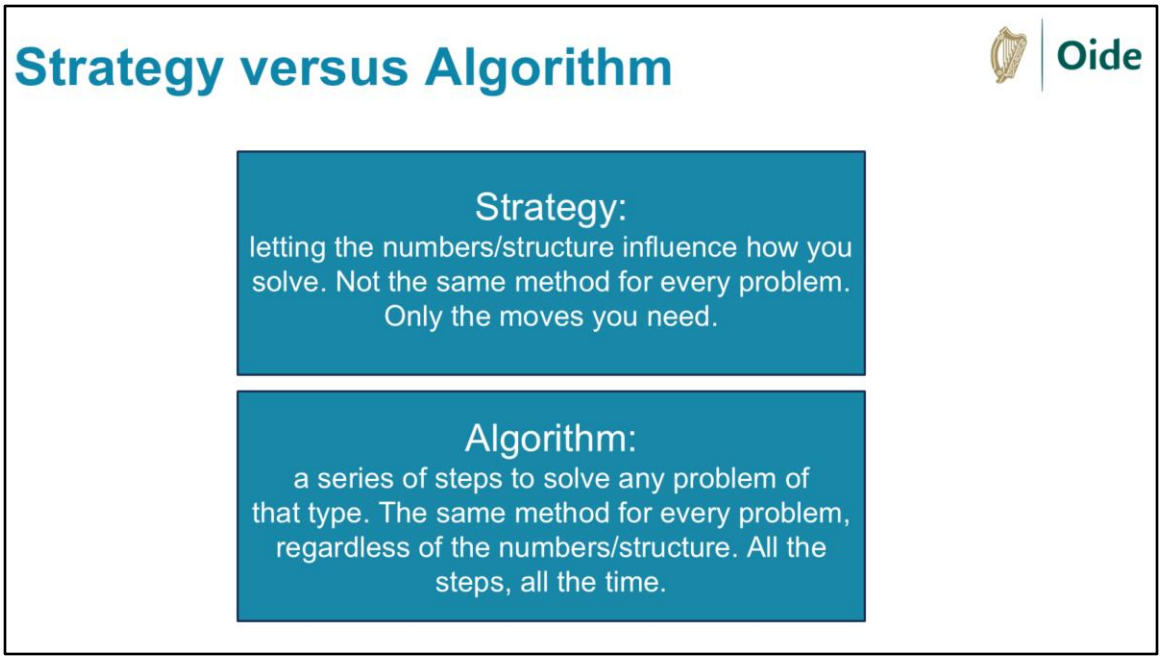
- Counting in doubles is also a great way for children to be flexible with numbers as they can then count forwards or backwards from the double.
- When children understand and can count flexibly in 2s, 5s and 10s, they can then break apart those numbers to count in 3s, 4s, 6s,7s, 8s, 9s. For example when children can count in 2s, they can then use this knowledge to count in 3s by doubling the number and adding one more. The same goes for counting in 5s, when children can count in 5s they can then count in 6s by multiplying the number by 5 and adding one more. The same works for counting in nines, when a student can count in 10s, they can multiply any number by 10 and subtract one away to find the answer for example 8 x 9= 8 x10 - 8

- We need children to understand the magnitude of number and see the patterns and relationship within the number sequence.
- As well as counting, another important aspect is learning about numerals - identify, recognise, sequence, order, locate and write numerals.
- Counting is for all stages and should be done daily.
- As students come to know the basic facts in any operation, they progress through three phases (Baroody, 2006)
    1. Counting
    2. Deriving (reasoning strategies on known facts)
    3. Mastery (efficiently produces answers)
- A counting session should have:
    - A lively pace.
    - Enthusiastic participation.
    - 2 or 3 short focused activities.
    - Physical activity.
    - Choral response.
    - Individual response.
- <u>Some Suggested Counting Activities: (</u>These activities can be done with any number range and when counting in single numbers or multiples)
    - Counting choir – start with choral counting, move onto parts of the choir.
    - Count around model bridging unusual ranges of numbers (forwards and backwards) 73 - 65
    - Counting Can (1s, 5s, 10s etc.)

- Counting stick - https://mathsbot.com/manipulatives/countingStick or https://bossmaths.com/countingstick/

Resource:

- Learning experiences that help to develop counting can be found here Counting Stage 3 and 4 https://pmc.oide.ie/resources/supportmaterialsforschools/

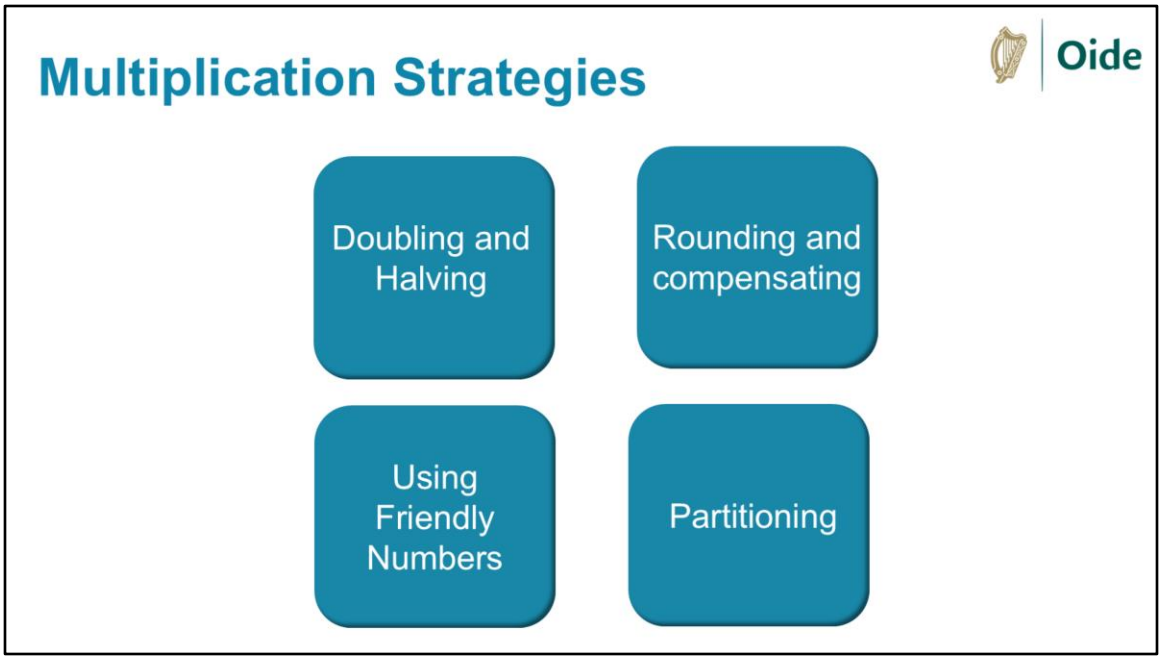


Purpose of slide:
Note any strengths/ benefits in choosing strategy vs algorithm.

Notes for teachers:

- A strategy is letting the numbers/structure influence how you solve a problem. The same method is not used for every problem. Only the moves you need are used.
- An algorithm is a series of steps to solve any problem of that type. The same method for every problem, regardless of the numbers/structure. All the steps, all the time.
- Algorithms provide a standardized, reliable way to solve problems, while strategies offer a flexible, understanding-based approach. Both are valuable tools in mathematics, but strategies encourage deeper thinking and problem-solving skills.
- Consider the benefits of having a developed number sense through counting, number work, multiple representations of number and number talks.

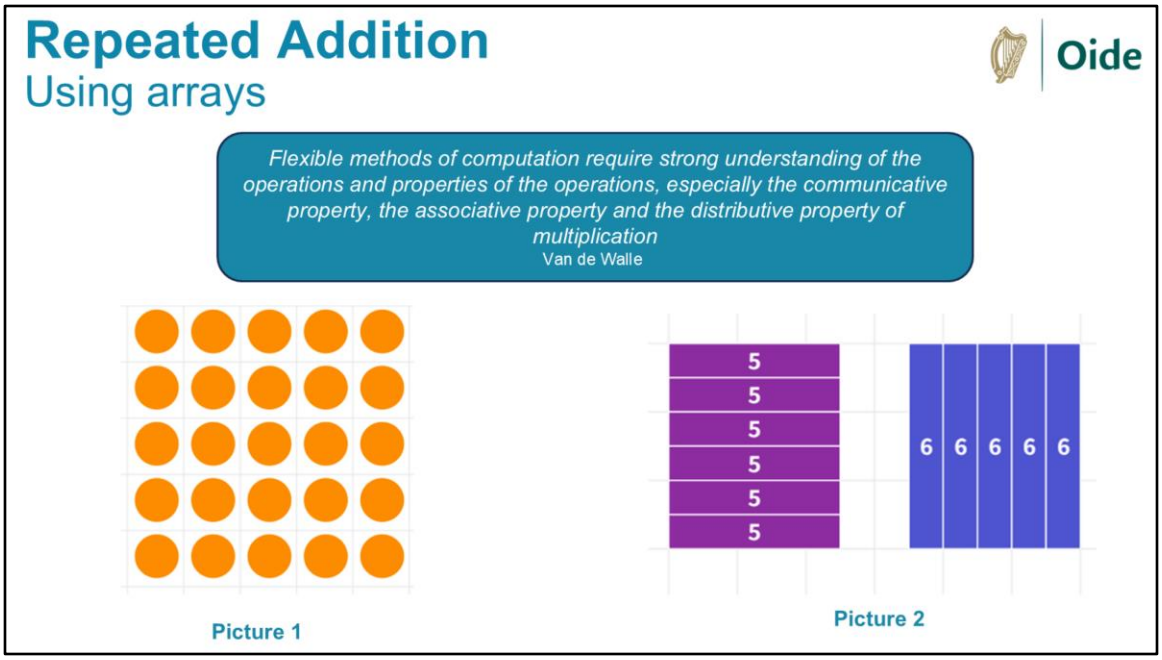
- It is important that children see connections, relationships and patterns using the strategies that make sense to them.
- Children require agency to be able to choose their methods, that make sense to them and not always have to rely on remembering the steps of the algorithm through rote learning.
- Number facts are important, but children need to be able to use them flexibly as they encounter various operational problems.



Purpose of slide:
To provide an overview of the major multiplication strategies.

Notes for teachers:

- There are 4 major multiplication strategies that children need to develop:
    - Doubling and Halving.
    - Rounding and compensating.
    - Using friendly numbers.
    - Partitioning.

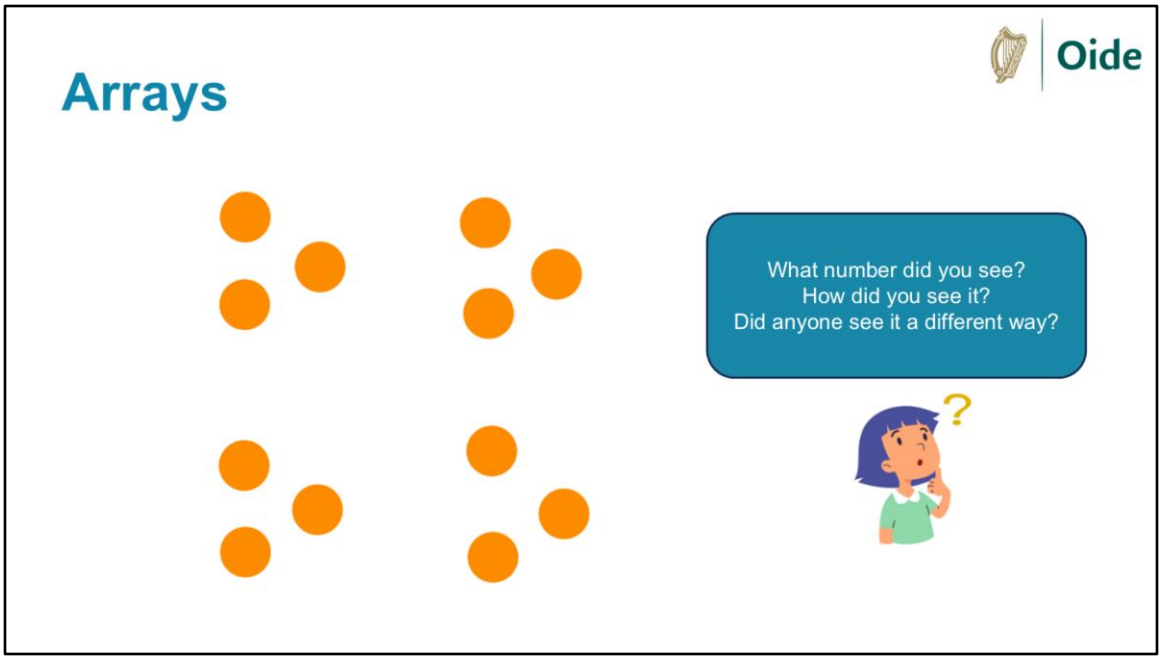


Purpose of slide:
Introducing Repeated Addition

Notes for teachers:

- Arrays are particularly useful representations of multiplication for several reasons, as follows:
    - Arrays can be quickly generated by drawing simple circles or X's
    - They help students make sense of the commutative property by simply turning them: 5 x 6 = 6 x 5
    - They help students see the relationship between multiplication and division. You can show the first row of an array of 8 for example: "How many circles are in the array if there are 8 counters in all?" This can help students see how 'something x 2=8' which is connected to 8 divided by 2.
    - They also lend themselves to break apart strategies for example 6 x 5 is the same as 5 x 5 and one more row.

- Picture one illustrates the commutative property using dot arrays. Children can build these arrays. For example, can you show me 5 x 6 in an array? What happens if you turn your array? This illustrates for children that multiplication is repeated addition. 5 rows of 6 counters or 6 rows of 5 counters. When children are accustomed to building arrays, the teacher can quickly flash up an array and ask "How many counters did you see? How did you see them?
- Picture 2 also illustrates the commutative property using Cuisenaire rods. Can you show me 5 x 6=6 x 5? As you can see, the 6 rods of 5 measures the same size as the 5 rods of 6.
- The first Maths Concept for stage 3 Sets and Operations mentions the commutative property: *"Commutative, associative, identity and distributive properties apply to the operation of multiplication.*"
- The second Maths concept is about multiplication having the same number of objects in each row. "*One definition of multiplication is having a certain number of groups of the same size. An early representation of multiplication is repeated addition".*

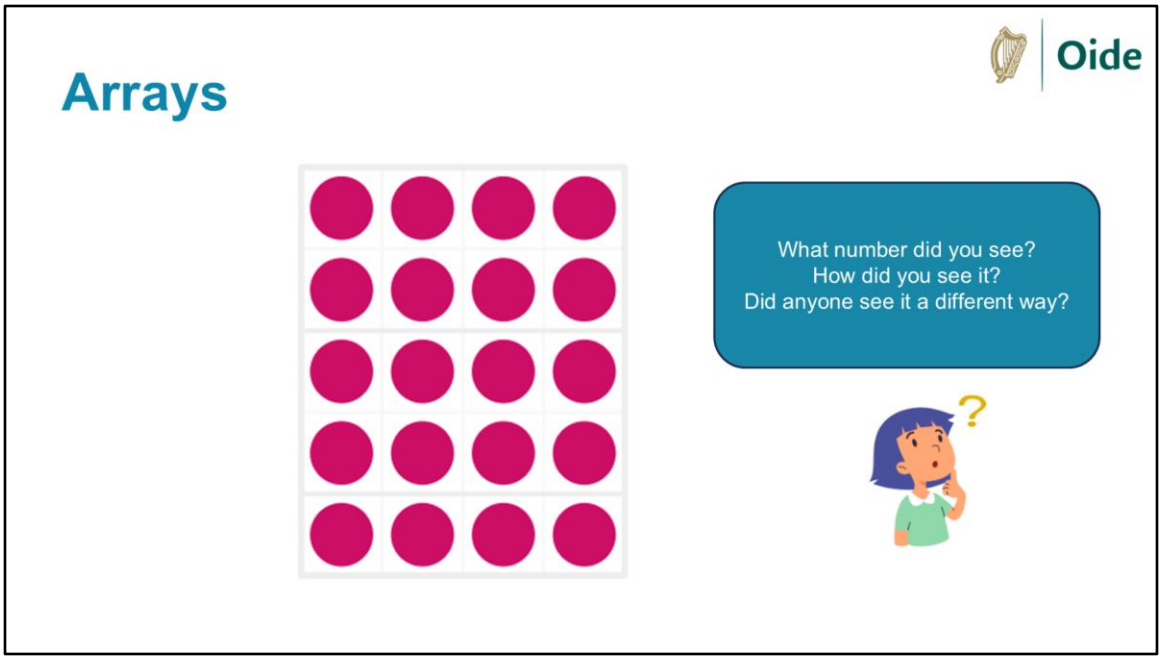


Purpose of slide:
To provide opportunities for pupils to explain how arrays make sense to them.

Notes for teachers:

- Show picture to pupils for 2 or 3 seconds, hide it, then show again for 2 or 3 seconds. Give the pupils time to share their thinking.
- Pose the questions: *What did you see? How did you see it? Did anyone see it in a different way.*
- It is important for pupils to get the opportunity to think aloud and discuss the variety of strategies as this can provide access to the task for other children who struggle with subitising.
- Sample answers might be:
    - I saw 3+3 which is 6 and I know that 6+6 is 12.
    - I know that 3 groups of 3 make 9 and one more is 12.

- I saw 4 groups of 3



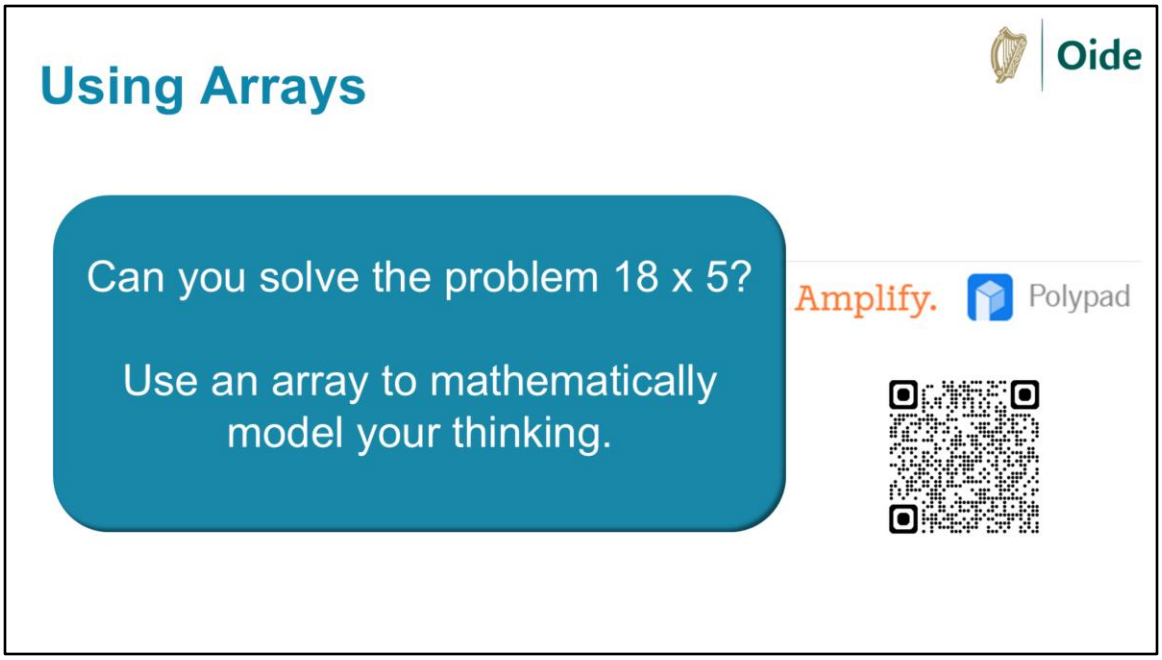
Purpose of slide:
Using a Structured Array.

Notes for teachers:

- Use the key questions as outlined on the slide above.
- Sample solutions:
    - I counted in two's.
    - I skip counted in 4s because there is 4 in each row.
    - I used doubling- I know that 4+4 is 8, then I added 8 and 8 to get 16 and one more row of 4 to make 20.
    - I split the array down the middle and counted 5 rows of 2 which I know is 10 and another 5 rows of 2 which is also 10 to make 20.

Resource:

- Ten frame on Amplify Poypad - https://mathigon.org/polypad/Hl4HaqAYXUVBqw



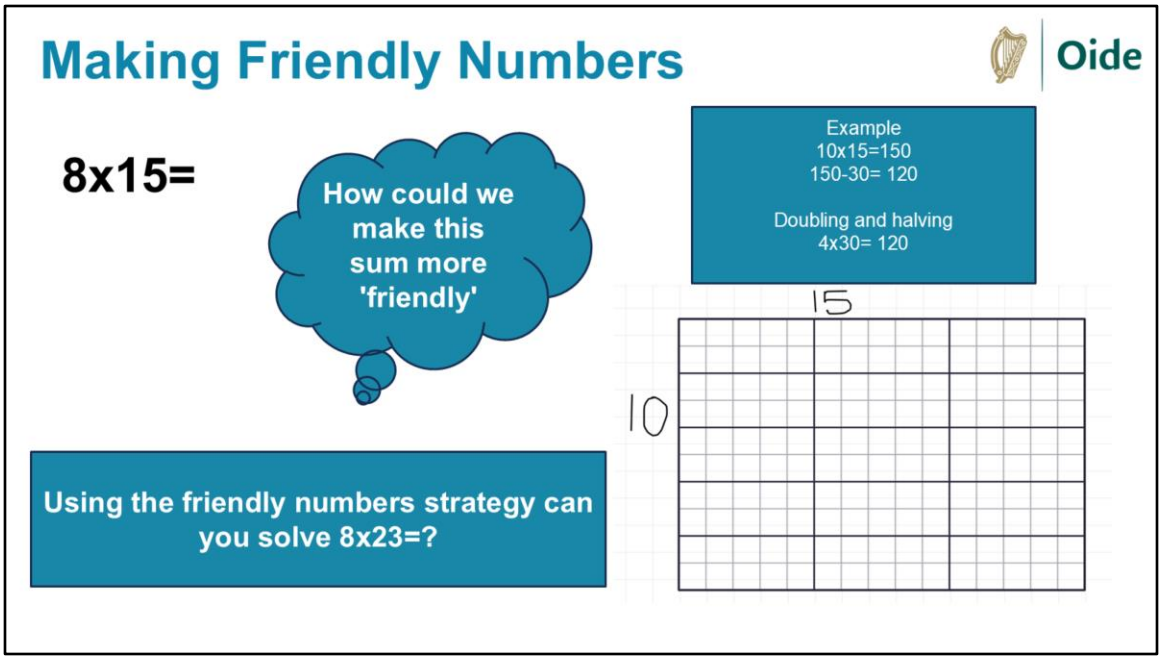
Purpose of slide:
Solve a problem using an array.

Notes for teachers:

- Consider the benefits of using an array in this case in place of the algorithm. (A mathematical model using manipulatives, drawings or digital tools can help teachers see a pupil's conceptual understanding around this problem)
- Use the QR code on the screen or the link below to take you to the Amplify Polypad.
- Can pupils make connections to other operations e.g. division from this model?

Resource:

- https://mathigon.org/polypad#number-frames

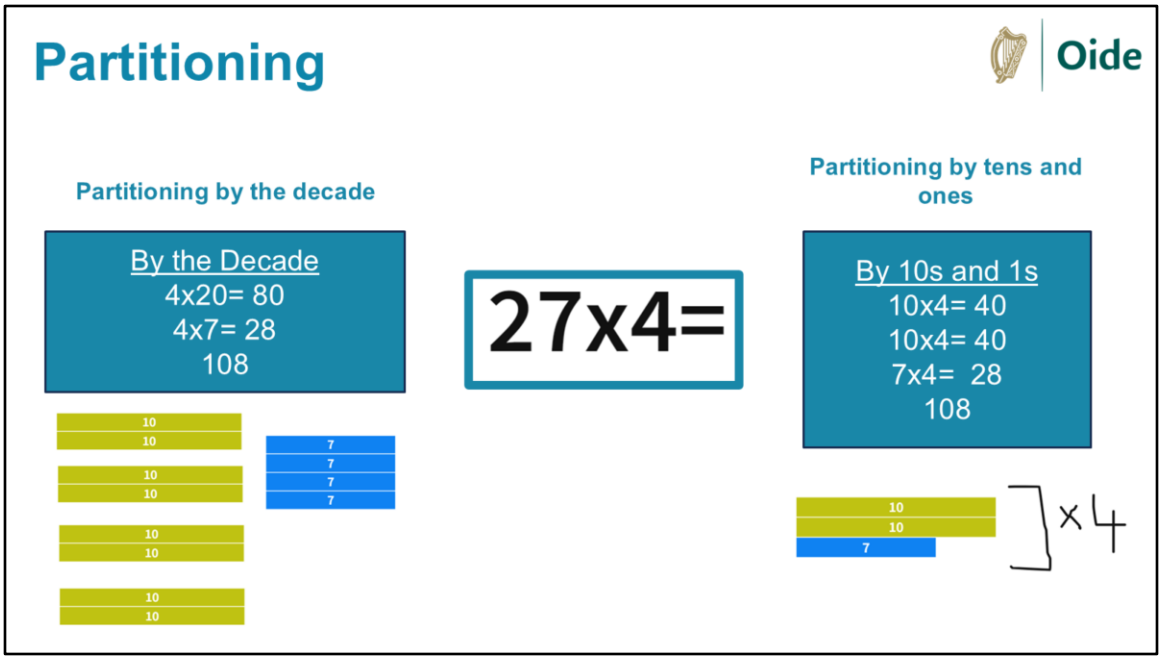


Purpose of slide:
Making and using friendly numbers.

Notes for teachers:

- Often a multiplication problem can be made easier by changing one of the factors to a friendly landmark number.
- 10 and 5 are friendly numbers. You are always only two numbers away from a friendly number e.g. 7 is only 2 away from 5; 8 is only 2 away from 10.
- Pupils who are comfortable multiplying by multiples of ten will often adjust factors to allow them to take advantage of this.
- This strategy is closely related to rounding and compensating, another multiplication strategy.
- Pupils might also use the doubling and halving strategy here - double 15 and half 8.

- Pupils will look at the numbers and consider which strategy is the most effective and efficient to use here.
- The image on the right of the slide is a sample of the area model. Here the student multiplies 10 x 15 - so 15 rows of ten. They can then cross off the two bottom rows to make 8 rows of 15 to find their answer.
- Try out 8 x 23 for yourself using the friendly number strategy and consider what this might look like in your class.



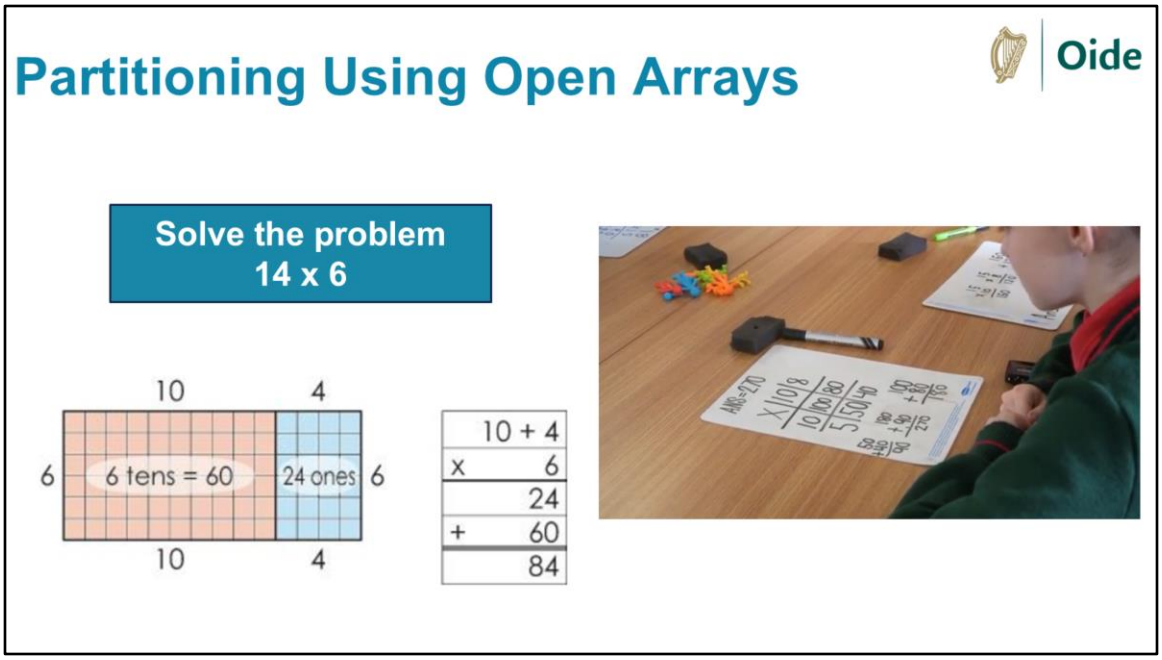
Purpose of slide:
To introduce the strategy of partitioning

Notes for teachers:

- Take the question 27 x 4. How would you solve it?
- All partitioning strategies are based on the distributive property of multiplication, mentioned in the maths concepts for Stage 3 Operations.
- Examples of how to use partitioning are examined here.
- Partitioning by the decade:(partition 27 into 20 and 7. Multiply each part by 4 and combine)
- Partitioning by 10s and 1s: (10x4) + (10x4) + (7x4) = 108
- Children break up numbers in a variety of ways that reflect their understanding of the base 10 concepts.
- By partitioning numbers to smaller numbers, children can flexibly solve more

complicated multiplication sums.

- Because students are breaking apart facts, a rectangle (using Cuisenaire rods) is an easier way to show this strategy rather than an array.



Purpose of slide:
Moving from using Cuisenaire style arrays to open arrays.

Notes for teachers:

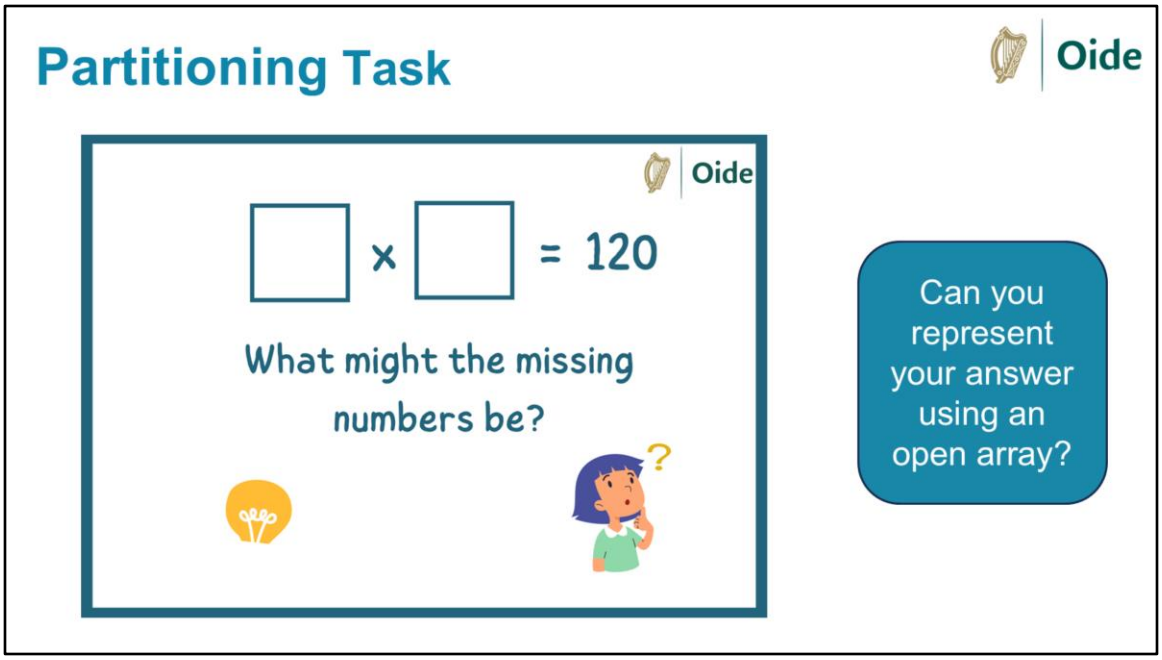
- Arrays not only help us to teach/understand multiplication, but they help us to relate multiplication seamlessly to division.
- Arrays help to support the shift from additive thinking (‘groups of ’ model) to multiplicative thinking (‘factor–factor–product’ model) (Siemon, 2013) and eventually to proportional and algebraic reasoning.
- The array model is a powerful way to help students understand not just the ‘how’ of multiplication but also the ‘why’. It supports students in a way which simply teaching the mechanics of the algorithm cannot.
- An Open array is also known as an Area Model.
- Using arrays to explore multiplication and division is a great foundation for the development of a more generalised area model.
- An area model is a rectangular diagram used in mathematics to solve multiplication problems, in which the factors being multiplied define the length and width of a rectangle. So the product is the area of the rectangle.

Hence, it is known as the “Area model for multiplication”. It links very well with the strand units of measuring and shape.

- Using the open array or area model can help pupils to model their thinking when multiplying numbers bigger than single digit numbers.
- Notice in the image on the left that the frames from Amplify Polypad are used to partition the sum into 10 x 6 + 4 x 6
- This is also demonstrated in the right image where an open array is used by the child to write the numerals and allow for the use of number facts in solving the sum.

Resource:

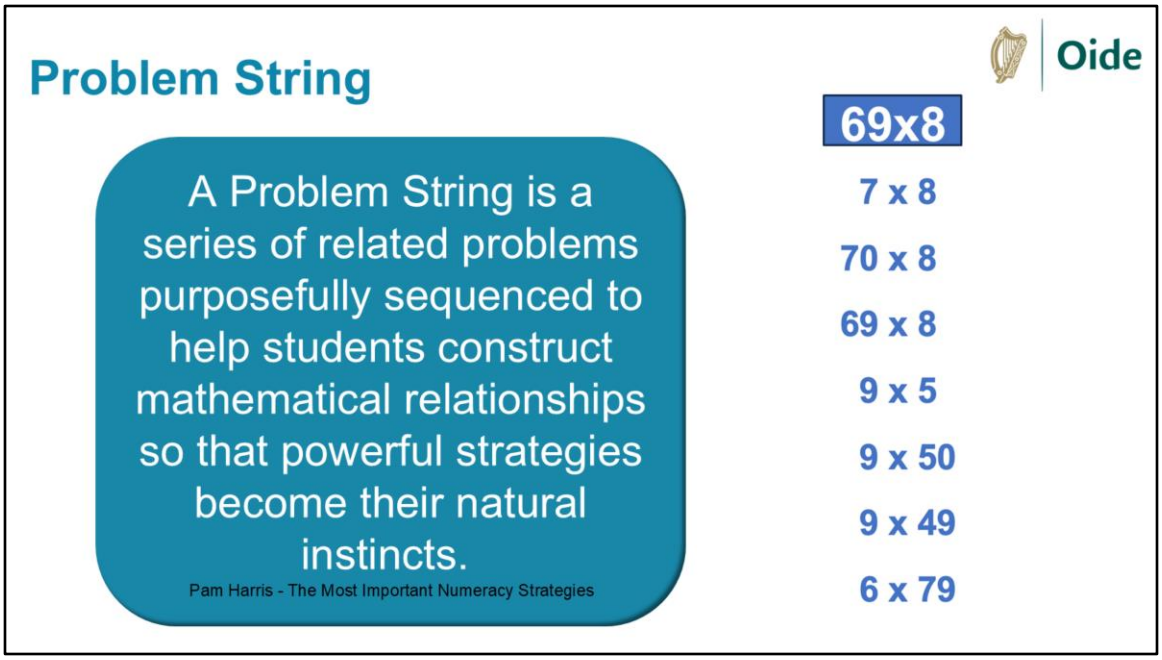
- The Area Model Multiplication on https://phet.colorado.edu/sims/html/area-model-multiplication/latest/area-model-multiplication_en.html is a great digital resource to model multiplication.



Purpose of slide:
To demonstrate an example of an open-ended task to practise partitioning.

Notes for teachers:

- Consider what prior knowledge children will need to engage in this task e.g. facts of ten, ten plus etc. Ten frames may be helpful to develop these.
- Children break up numbers in a variety of ways that reflect their understanding of the base 10 concepts.
- By partitioning numbers to smaller numbers, children can flexibly solve more complicated multiplication sums.
- How might the children solve this? They could partition by decades, 10s and ones and partition the multiplier- this strategy works only for multiplication.
- How will their array look? They could draw it, use Cuisenaire rods or other digital tools.



Purpose of slide:
To introduce the use of problem strings.

Notes for teachers:

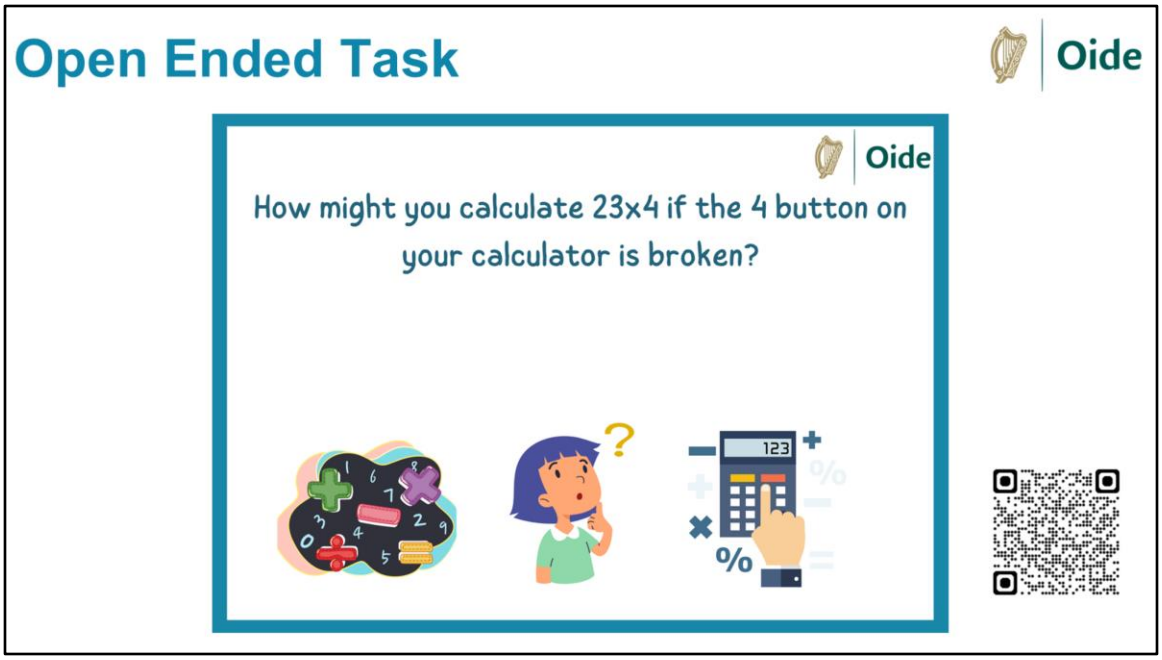
- A Problem String is a powerful teaching tool based on the idea that learning mathematics is about constructing relationships and connections.
- A Problem String is a series of related problems purposefully sequenced to help pupils construct mathematical relationships so that powerful strategies become their natural instincts.
- This powerful teaching tool is *designed* to help pupils mentally construct mathematical relationships.
- In this mini-lesson structure, teachers and pupils interact to construct important mathematical strategies, models, and concepts.
- The power of a Problem String lies in the carefully crafted conversation as pupils solve problems, one at a time, and the teacher makes pupils thinking

visible and draws out important connections and relationships.

- Pam Harris Podcast '*Maths is Figure-out-able'* is also a great resource to learn more about problem strings.
- How it might look and sound like in the classroom.
    - What is 7 x 8? Give pupils time to think. Model-represent on a number line. “Even though none of you are writing, I’ll just show 7x8 on a number line”.
    - What is 70 x 8? Give pupils time to think. “How do you know? Did anyone use the problem before, 7 x 8, to help them? How? Represent using 7 x 8 to get 70 x 8 using another strategy.
    - What is 69 x 8? Give pupils time to think. “What is 69 x 8? How do you know? Did anyone use the problem, 70 x 8, to help them? How? ”

Resource:

- For further information about Problem Strings you might like to visit https://www.mathisfigureoutable.com/blog/what-is-the-difference-between-a-problem-talk-and-a-problem-string
- Pam Harris' Podcast - Math is Figureoutable Episode 71: "What are Problem Strings Anyway?" provides more insights on this method.



Purpose of slide:
To show an open-ended task where the pupils can use different multiplication strategies.

Notes for teachers:

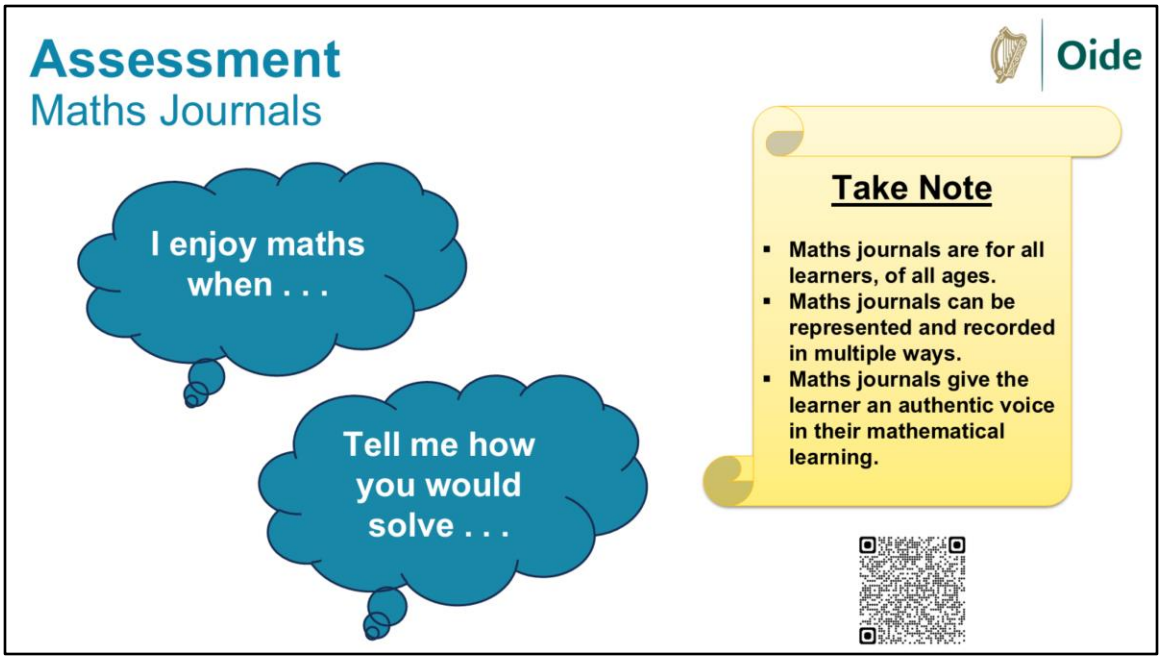
- An open-ended task is one where there is a range of 'correct' solutions and/or a range of ways to achieve one or more solutions.
- Using open-ended tasks is one way to encourage playfulness in mathematics and foster a productive disposition to mathematics.
- Open-ended tasks, like this task, provide opportunities for *exploration, investigation, challenge, creativity, choice and independence.*
- Other examples of open-ended tasks will be shared with participants.
- A key aspect of children engaging in open-ended tasks is the follow-up discussions that take place either in a small group or a whole class setting.
- If the emphasis is placed on the generation of different ideas, all children feel that they have something to contribute, and, moreover, learn from the ideas and strategies of their peers.
- Open-ended tasks are also one way of providing for cognitively

challenging tasks in maths.

- When selecting an open-ended task, as with selecting any task, it is important to keep in mind the mathematical point.
- Model talk moves during the follow up discussion e.g. tell me more, can you repeat what …. said.

Resource:

Other Open-Ended tasks are available on https://pmc.oide.ie/resources/micro-maths/ or via the QR code



Purpose of slide:
To provide reflective prompts for use in class.

Notes for teachers:

- Journals are useful for both teachers and learners to assess attitudes, knowledge and skills.
- Children can keep track of their thinking and understanding in the journal.
- Journals can contain general observations about Maths or can be more specific and focus on a particular concept.
- On the slide are two journal prompts which can be used in class.
- Journal prompts -
  - I enjoy Maths when .... general prompt to get the children to express their feelings towards maths.
  - Tell me how you would solve .... e.g. 10 x 5, 23 x 4 (teacher can choose their own problem so that children can demonstrate different multiplication strategies).
- A Maths Journal encourages a child to:
  - Reflect on what they have learned and put it in their own words.
  - Discuss maths with others (pupil and teacher).

- Identify areas of strength and weakness.
- Evaluate what they have learned.
- If journal writing is done on a regular basis it will help promote mathematical understanding.
- Writing about Maths can have a positive effect that on reducing pupil's Maths anxiety.

Resource:
Further information about Maths Journals available via:
https://pmc.oide.ie/resources/supportmaterialsforschools/